CHAPTER 3

NODAL AND MESH ANALYSIS

Problem 3.1

Apply nodal analysis to find V_1 and V_2 . Then, find the current I that passes through the 2Ω resistance.

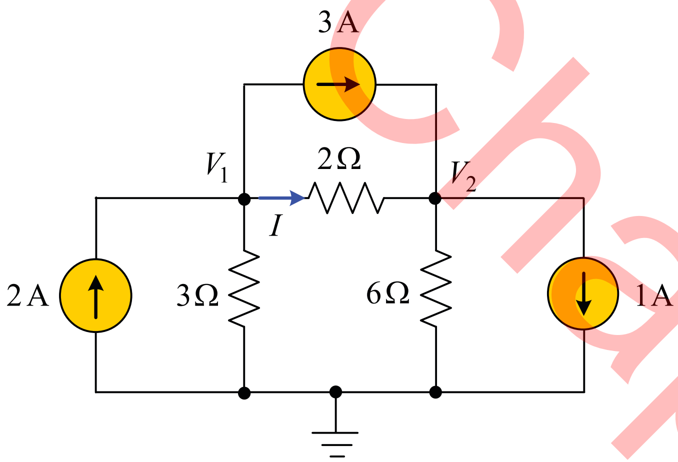

Figure 1

Solution

As it is shown in Fig. 2, we define arbitrarily the directions of current I_1 and I_2. The nodal equations have the following form

node V_1: $I + I_1 + 3 = 2 \Rightarrow I + I_1 = -1$

node V_2: $I+3=I_2+1 \Rightarrow I-I_2=-2$

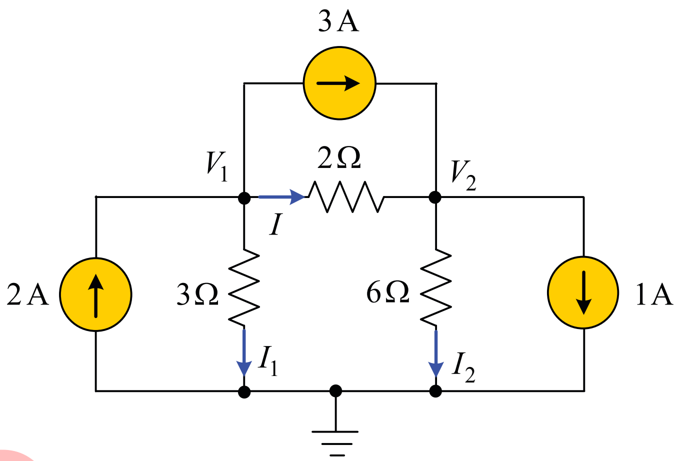

Figure 2

The currents as functions of the node voltages are given by

$$I=\frac{V_1-V_2}{2}$$

$$I_1=\frac{V_1}{3}$$

$$I_2=\frac{V_2}{6}$$

Now, we substitute the current in the node equations:

$$\frac{V_1-V_2}{2}+\frac{V_1}{3}=-1$$

$$\frac{V_1-V_2}{2}-\frac{V_2}{6}=-2$$

or

$$5V_1-3V_2=-6$$

$$-3V_1+4V_2=12$$

Thus, we obtain a linear system of two equations, that is symmetric in relative to the main diagonal. The solution of this system gives the following values

$$V_1 = 1.091\,\text{V} \;\text{ and }\; V_2 = 3.818\,\text{V}$$

Thus,

$$I = \frac{V_1 - V_2}{2} = -1.364\,\text{A}$$

Problem 3.2

Apply nodal analysis to find voltage V_I and current I_V.

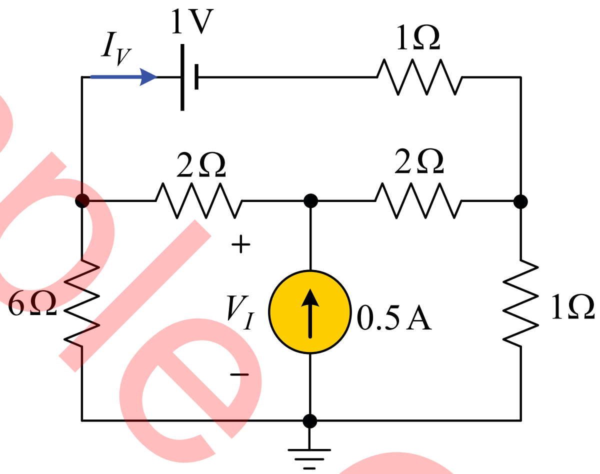

Figure 1

Solution

In the circuit of Fig. 2, the application of the KCL leads to the following nodal equations:

node a: $I_1 + I_2 + I_V = 0$

node b: $I_1 + 0.5 = I_3$

node c: $I_3 + I_4 + I_v = 0$

The branch currents can be expressed as

$$I_1 = \frac{V_a - V_b}{2}$$

$$I_2 = \frac{V_a}{6}$$

$$I_3 = \frac{V_b - V_c}{2}$$

$$I_4 = \frac{-V_c}{1}$$

$$I_V = \frac{V_a - 1 - V_c}{1} = V_a - V_c - 1$$

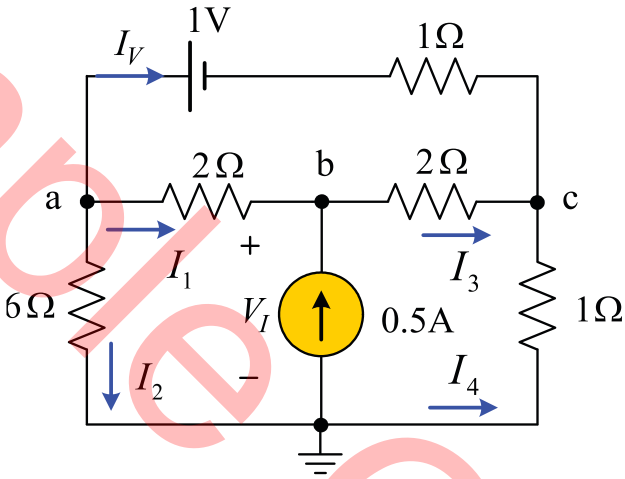

Figure 2

Substituting to the nodal equations we get

$$\frac{V_a - V_b}{2} + \frac{V_a}{6} + V_a - V_c - 1 = 0$$

$$\frac{V_a - V_b}{2} + 0.5 = \frac{V_b - V_c}{2}$$

$$\frac{V_b - V_c}{2} + V_a - 2V_c = 1$$

or

$$10V_a - 3V_b - 6V_c = 6$$

$$V_a - 2V_b + V_c = -1$$

$$2V_a + V_b - 5V_c = 2$$

The solution of this system gives the following values

$$V_a = 1.154\,\text{V},\ \ V_b = 1.231\,\text{V}\ \text{ and }\ V_c = 0.308\,\text{V}$$

Consequently

$$V_I = V_b = 1.231\,\text{V}$$

and

$$I_V = V_a - V_c - 1 = -0.154\,\text{A}$$

Problem 3.3

Find current I_1.

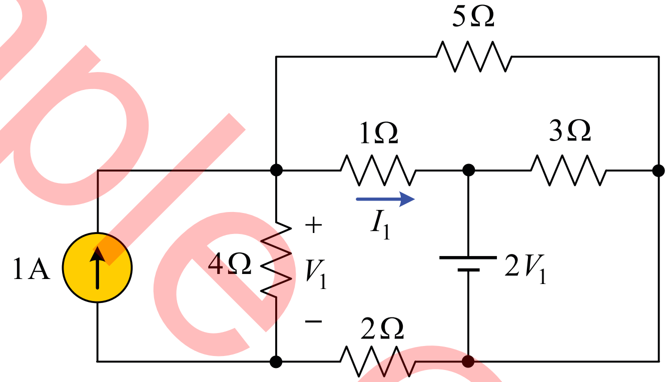

Figure 1

Solution

We define, as the reference node, the negative terminal of the voltage source. As it is shown in Fig. 2, the voltages at nodes c and d are known. Therefore, we must obtain the nodal equations for the other two nodes a and b:

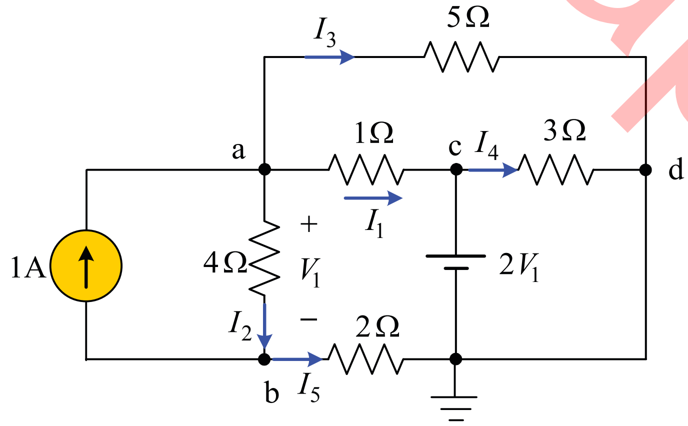

Figure 2

node a: $I_1 + I_2 + I_3 = 1$

node b: $I_2 = 1 + I_5$

Also

$$V_a - V_b = V_1$$

$$V_c = 2V_1 = 2V_a - 2V_b$$

The currents can be expressed as

$$I_1 = V_a - V_c = V_a - (2V_a - 2V_b) = -V_a + 2V_b$$

$$I_2 = \frac{V_a - V_b}{4}$$

$$I_3 = \frac{V_a - V_d}{5} = \frac{V_a}{5}$$

$$I_4 = \frac{V_c - V_d}{3} = \frac{2V_a - 2V_b}{3}$$

$$I_5 = \frac{V_b}{2}$$

Substituting the currents in the nodal equations we get

$$-V_a + 2V_b + \frac{V_a - V_b}{4} + \frac{V_a}{5} = 1 \Rightarrow -11V_a + 35V_b = 20$$

$$\frac{V_a - V_b}{4} = 1 + \frac{V_b}{2} \Rightarrow V_a - 3V_b = 4$$

Consequently

$$V_a = \frac{\begin{bmatrix} 20 & 35 \\ 4 & -3 \end{bmatrix}}{\begin{bmatrix} -11 & 35 \\ 1 & -3 \end{bmatrix}} = \frac{-60 - 140}{33 - 35} = \frac{-200}{-2} = 100\,\text{V}$$

$$V_b = \frac{\begin{bmatrix} -11 & 20 \\ 1 & 4 \end{bmatrix}}{\begin{bmatrix} -11 & 35 \\ 1 & -3 \end{bmatrix}} = \frac{-44 - 20}{33 - 35} = \frac{-64}{-2} = 32\,\text{V}$$

Thus,

$$I_1 = -V_a + 2V_b = -100 + 64 = -36\,\text{A}$$

Problem 3.4

For the circuit in Fig. 1, find (as a function of the circuit elements) the voltages at nodes a and b.

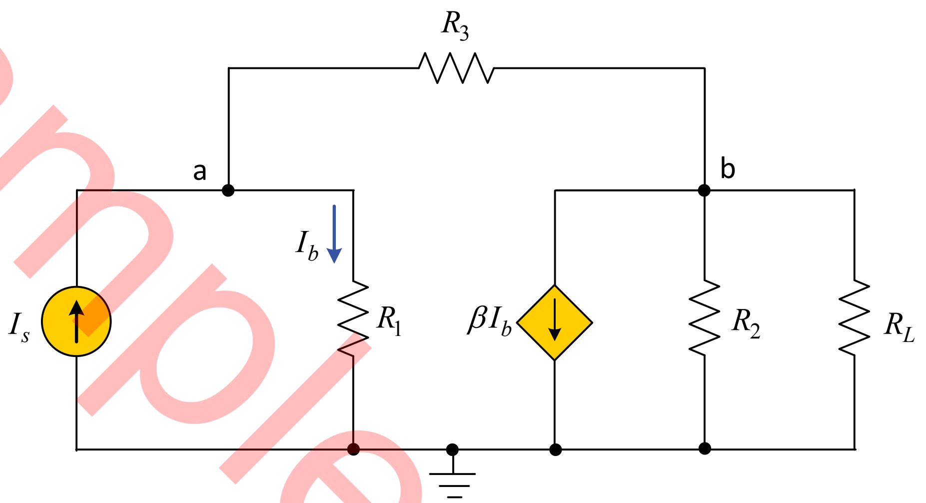

Figure 1

Solution

We define the currents as it is shown in the circuit in Fig. 2 and we get the equations at nodes a and b.

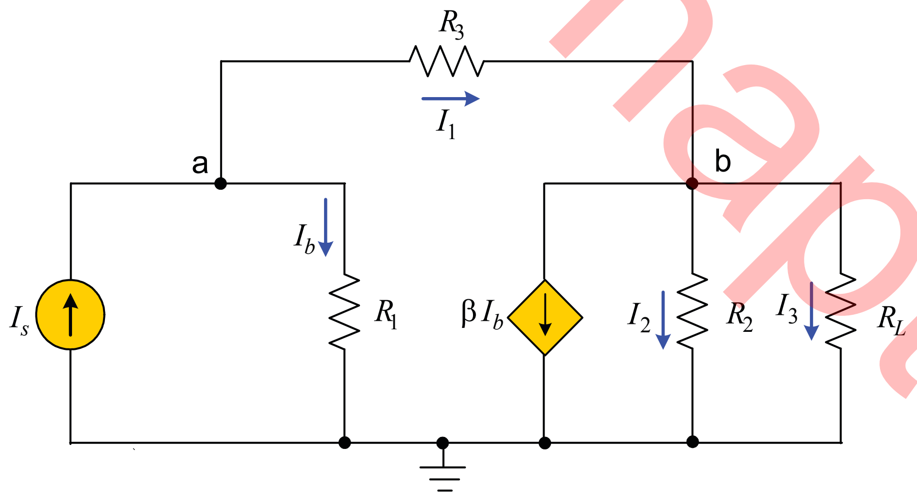

Figure 2

node a: $I_s = I_1 + I_b$

node b: $I_1 = I_2 + I_3 + \beta I_b$

The currents are equal to

$$I_b = \frac{E_a}{R_1},$$

$$I_1 = \frac{E_a - E_b}{R_3},$$

$$I_2 = \frac{E_b}{R_2}, \text{ and}$$

$$I_3 = \frac{E_b}{R_L}$$

Substituting the currents in the nodal equations we get

$$I_s = \frac{E_a - E_b}{R_3} + \frac{E_a}{R_1}$$

$$\frac{E_a - E_b}{R_3} = \frac{E_b}{R_2} + \beta\frac{E_a}{R_1} + \frac{E_b}{R_L}$$

or

$$\left(\frac{1}{R_1} + \frac{1}{R_3}\right)E_a - \frac{1}{R_3}E_b = I_s$$

$$\left(\frac{\beta}{R_1} - \frac{1}{R_3}\right)E_a + \left(\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_L}\right)E_b = 0$$

The solution of the above system gives

$$E_a = \frac{R_1(R_3R_L + R_2R_L + R_2R_3)I_s}{R_3R_L + R_2R_L + R_2R_3 + R_1R_L + R_1R_2 + R_2R_Lb}$$

and

$$E_b = \frac{R_2R_L(R_1 - bR_3)I_s}{R_3R_L + R_2R_L + R_2R_3 + R_1R_L + R_1R_2 + R_2R_Lb}$$

Problem 3.5

Find I in Fig. 1 using mesh analysis.

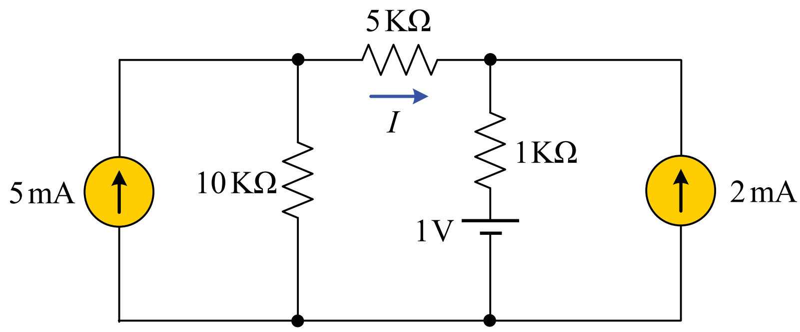

Figure 1

Solution

We define the mesh currents as shown in the circuit of Fig. 2. We have two obvious equations, i.e. $J_1 = 5$ mA and $J_3 = -2$ mA.

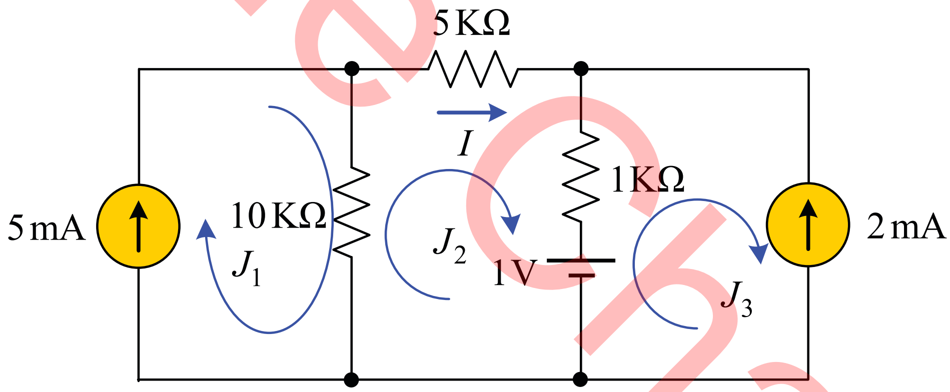

Figure 2

The equation for the second loop has the form

$$-10000J_1 + 16000J_2 - 1000J_3 = -1$$

Substituting the current values we finally get

$$-50 + 16000J_2 + 2 = -1$$

$$\Rightarrow J_2 = I = \frac{47}{16} \text{ mA}$$

Problem 3.6

Find I in Fig. 1 using mesh analysis.

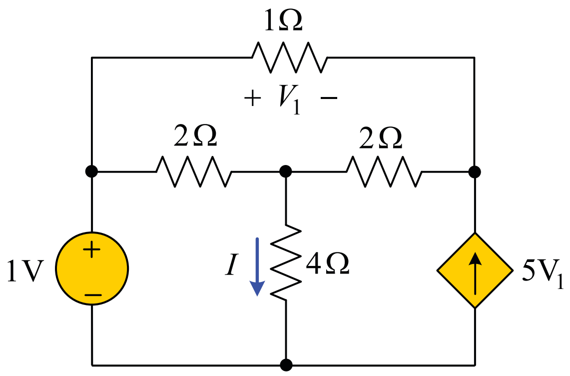

Figure 1

Solution

We define the mesh currents as in Fig. 2. We observe that

$$J_2 = -5V_1$$

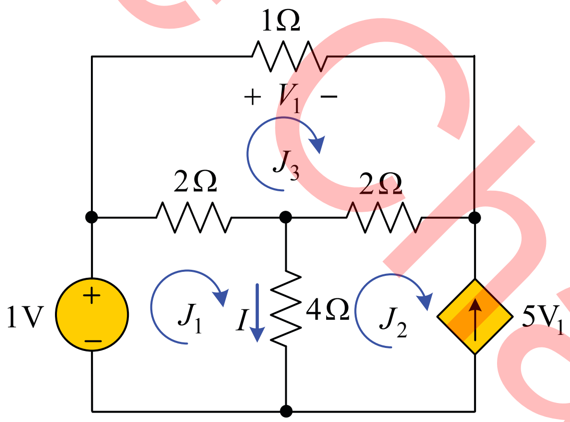

Figure 2

Then, as a second step, we apply the KVL to mesh 1 and 3:

$$6J_1 - 4J_2 - 2J_3 = 1$$

$$-2J_1 - 2J_2 + 5J_3 = 0$$

However

$$V_1 = J_3 \Rightarrow J_2 = -5J_3$$

Substituting the value of J_2 we lead to the following linear system

$$6J_1 + 18J_3 = 1$$

$$-2J_1 + 15J_3 = 0$$

whose solution gives

$$J_1 = \frac{5}{42}\text{A} \quad \text{and} \quad J_3 = \frac{1}{63}\text{A}$$

Therefore

$$J_2 = -5J_3 = -\frac{5}{63}\text{A}$$

and

$$I = J_1 - J_2 = \frac{5}{42} + \frac{5}{63} = 0.198\,\text{A}$$

Problem 3.7

Calculate the voltage V_a.

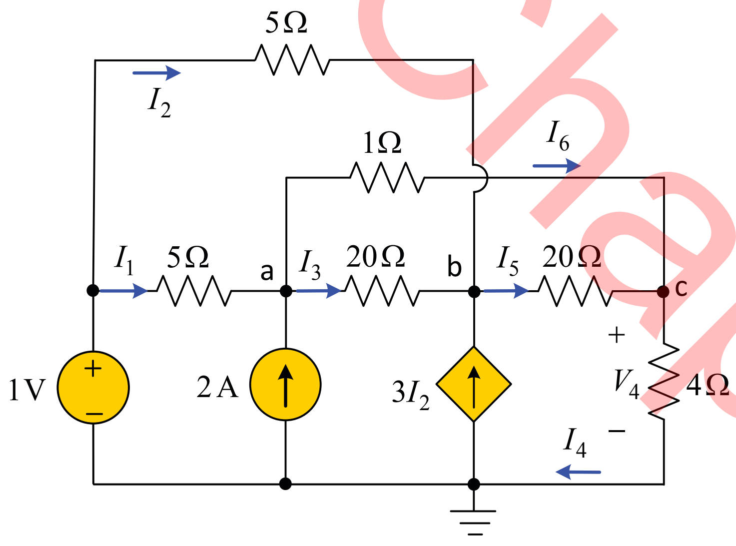

Figure 1

Solution

A node has an obvious voltage of 1V. The equations at nodes a, b and c are

node a:

$$I_1 + 2 = I_3 + I_6$$

$$\Rightarrow \frac{1 - V_a}{5} + 2 = \frac{V_a - V_b}{20} + \frac{V_a - V_c}{1}$$

node b:

$$3I_2 + I_3 + I_2 = I_5$$

$$\Rightarrow 4\frac{1 - V_b}{5} + \frac{V_a - V_b}{20} = \frac{V_b - V_c}{20}$$

node c:

$$I_5 + I_6 = I_4$$

$$\Rightarrow \frac{V_b - V_c}{20} + V_a - V_c = \frac{V_c}{4}$$

From the above relations we obtain the following system of linear equations

$$25V_a - V_b - 20V_c = 44$$

$$-V_a + 18V_b - V_c = 16$$

$$-20V_a - V_b + 26V_c = 0$$

The solution of this system gives

$$V_a = 4.827\,\text{V}$$

$$V_b = 1.366\,\text{V}$$

$$V_c = 3.766\,\text{V}$$

Problem 3.8

Find current I_x and the node voltages.

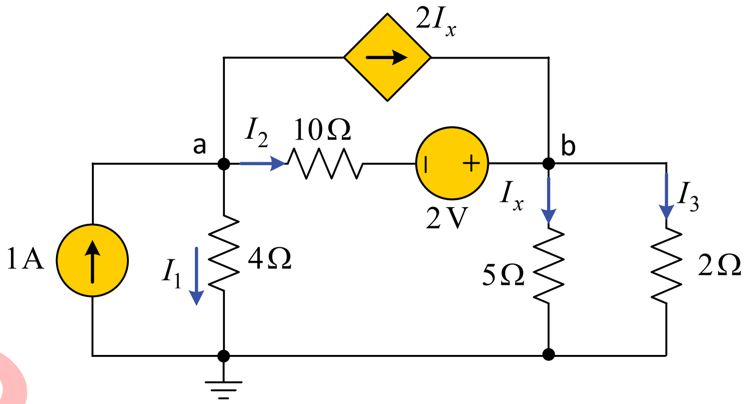

Figure 1

Solution

The two nodal equations are

node a: $$\frac{V_a}{4}+\frac{V_a+2-V_b}{10}+2I_x=1$$

node b: $$2I_x+\frac{V_a+2-V_b}{10}=I_x+\frac{V_b}{2}$$

At node b, the voltage is equal to

$$V_b=5I_x$$

Substituting V_b we get the system

$$7V_a+30I_x=16$$

$$20I_x-V_a=2$$

The solution of the above system gives

$$I_x=0.176\,\text{A} \quad \text{and} \quad V_a=1.529\,\text{V}$$

Therefore

$$V_b=5I_x=0.882\,\text{V}$$

Problem 3.9

Find current *I*.

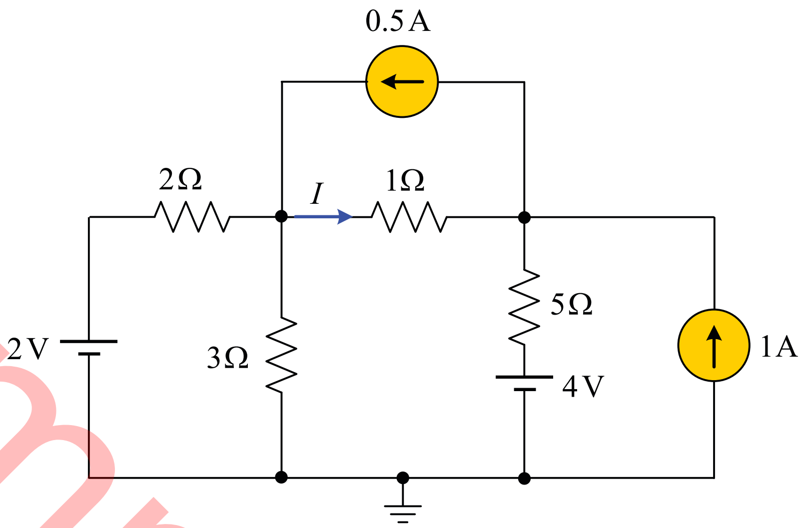

Figure 1

Solution

We have two current sources. Therefore, it is preferable to apply mesh analysis with obvious equations. Specifically, we observe that

$$J_3 = -0.5\,\text{A} \quad \text{and} \quad J_4 = -1.0\,\text{A}$$

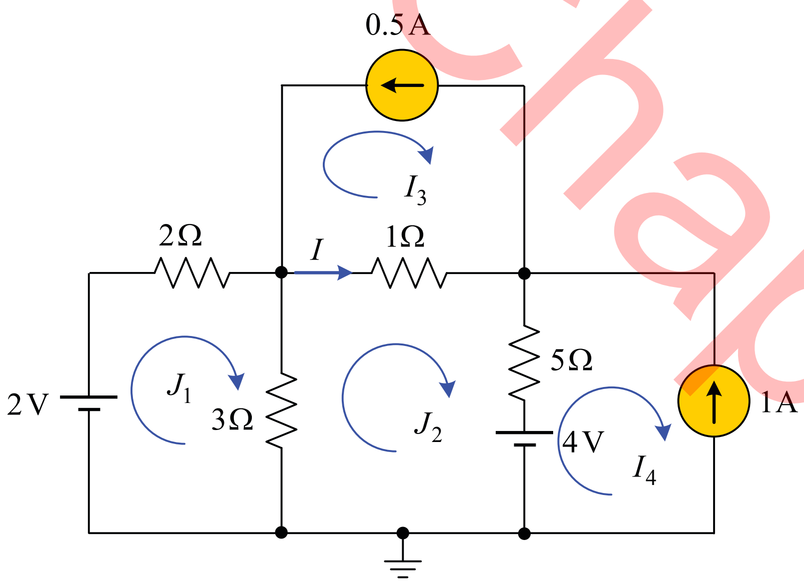

Figure 2

Writing KVL for mesh 1 and mesh 2, we obtain

$$5J_1 - 3J_2 = 2$$

$$-3J_1 + 9J_2 - J_3 - 5J_4 = -4 \Rightarrow -3J_1 + 9J_2 = -9.5$$

The solution of the above system of linear equations gives

$$J_1 = -0.292\,\text{A} \quad \text{and} \quad J_2 = -1.153\,\text{A}$$

Therefore

$$I = J_2 - J_3 = -0.653\,\text{A}$$

Problem 3.10

Use nodal analysis to find current I_1.

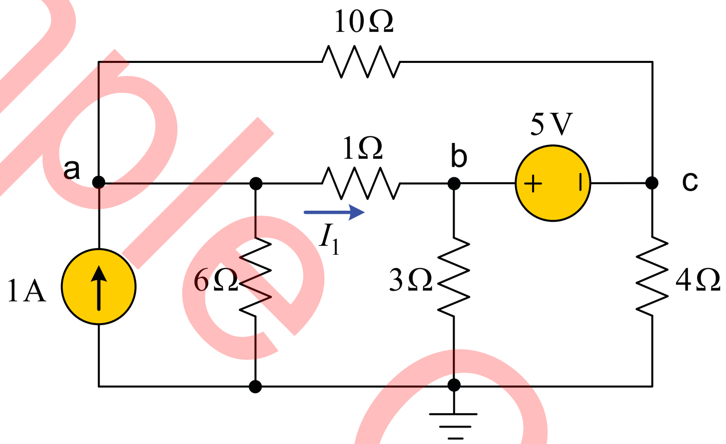

Figure 1

Solution

As we can see in Fig. 2, there is a supernode that includes nodes b and c. The equations at supernode and at node a, are

supernode:

$$I_1 + I_3 = I_4 + I_5 \qquad (1)$$

node a:

$$I_1 + I_2 + I_3 = 1 \qquad (2)$$

From the supernode we also have the following equation

$$V_b - V_c = 5 \qquad (3)$$

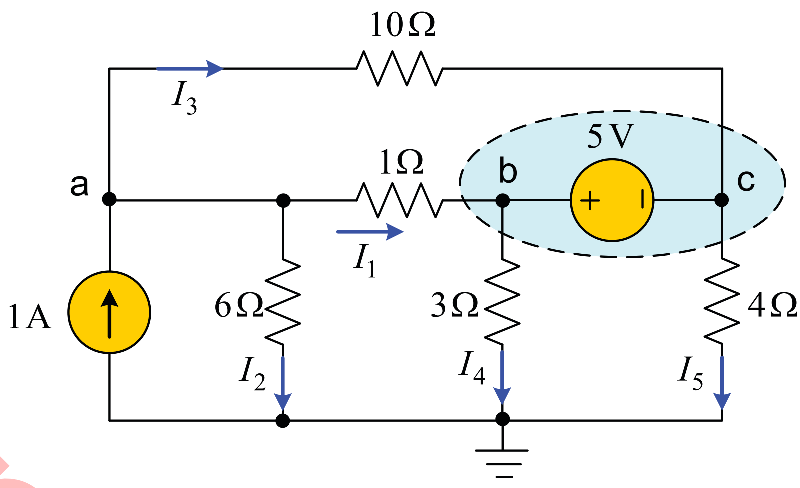

Figure 2

The node currents can be expressed as

$$I_1 = V_a - V_b \tag{4}$$

$$I_2 = \frac{V_a}{6} \tag{5}$$

$$I_3 = \frac{V_a - V_c}{10} \tag{6}$$

$$I_4 = \frac{V_b}{3} \tag{7}$$

$$I_5 = \frac{V_c}{4} \tag{8}$$

Substituting the above relations in Eqs. (1) and (2) we get

$$V_a - V_b + \frac{V_a - V_c}{10} = \frac{V_b}{3} + \frac{V_c}{4} \tag{9}$$

$$V_a - V_b + \frac{V_a}{6} + \frac{V_a - V_c}{10} = 1 \tag{10}$$

or

$$33V_a - 40V_b - 10.5V_c = 0 \tag{11}$$

$$76V_a - 60V_b - 6V_c = 60 \tag{12}$$

Eq. (3), (11) and (12) form a system of three equations with three unknowns. The solution of this system gives

$$V_a = V_b = 3\,\text{V} \ \text{ and } \ V_c = -2\,\text{V} \tag{13}$$

Therefore

$$I_1 = V_a - V_b = 0\,\text{A} \tag{14}$$

Problem 3.11

For the circuit in Fig. 1 find I if $V_2 = 0$.

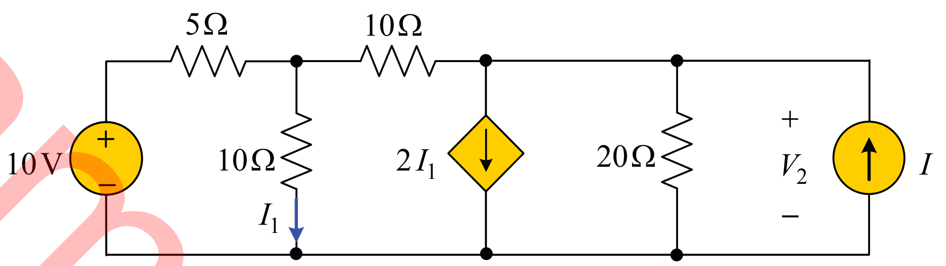

Figure 1

Solution

In the circuits of Fig. 2, we have the following equation at nodes V_1 and V_2:

$$\frac{V_1 - 10}{5} + \frac{V_1}{10} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{20} + 2I_1 = I$$

However

$$V_2 = 0\,\text{V}$$

and

$$I_1 = \frac{V_1}{10}$$

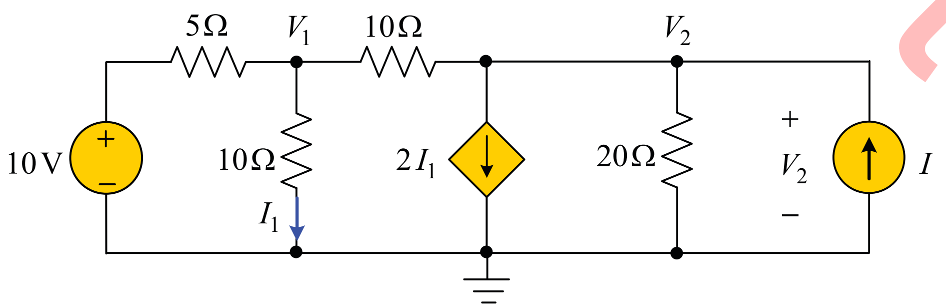

Figure 2

Therefore

$$\frac{V_1-10}{5}+\frac{V_1}{10}+\frac{V_1}{10}=0\Rightarrow V_1=5\,\text{V}$$

and

$$\frac{V_2-V_1}{10}+\frac{V_2}{20}+2I_1=I$$

$$\Rightarrow\frac{-V_1}{10}+2\frac{V_1}{10}=I\Rightarrow I=\frac{V_1}{10}=0.5\,\text{A}$$

Problem 3.12

Use mesh analysis to find the voltage V_x.

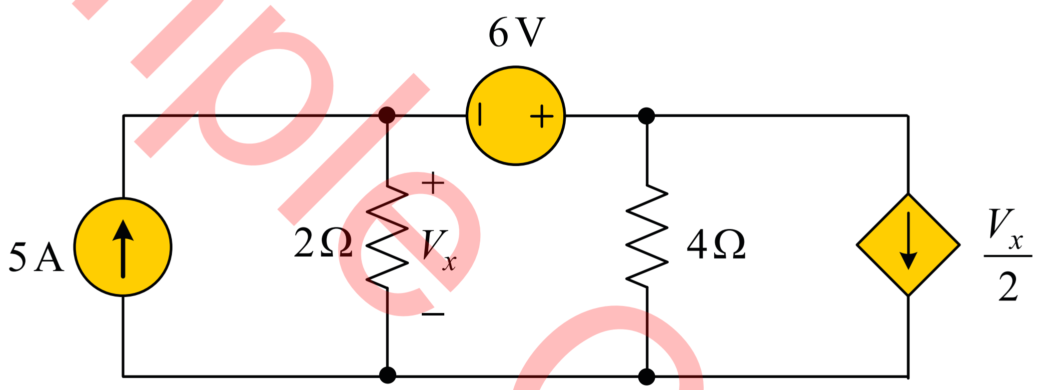

Figure 1

Solution

Let as define the currents loops as in Fig. 2.

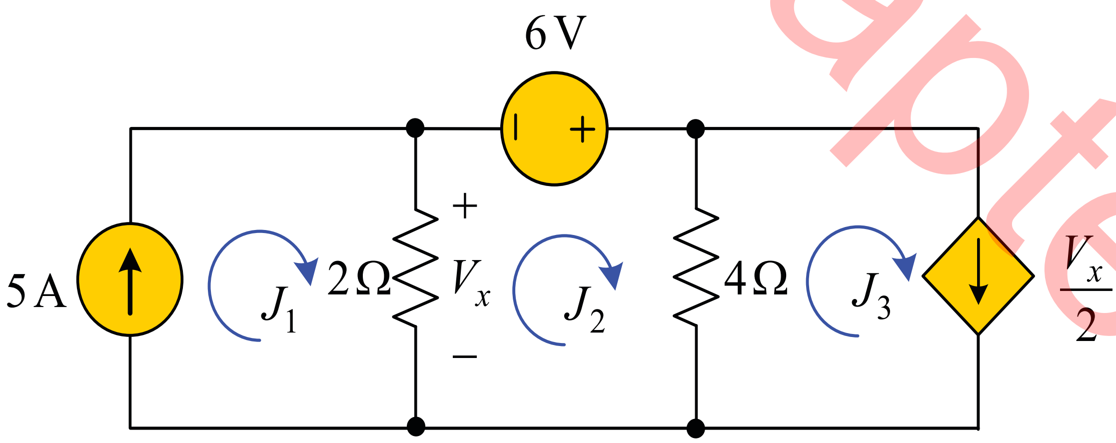

Figure 2

We have the following obvious equations

$$J_1 = 5\,\text{A} \quad \text{and} \quad J_3 = \frac{V_x}{2}$$

The equation in the second loop is

$$-2J_1 + 6J_2 - 4J_3 = 6$$

Substituting the current J_1 we obtain

$$-10 + 6J_2 - 2V_x = 6$$

$$\Rightarrow J_2 = \frac{8 + V_x}{3}$$

However, in the 2Ω resistor we have

$$V_x = 2(J_1 - J_2) = 10 - 2J_2$$

Therefore

$$J_2 = \frac{8 + 10 - 2J_2}{3} \Rightarrow J_2 = \frac{18}{5}\,\text{A}$$

Thus,

$$V_x = 10 - 2J_2 = 10 - 2\frac{18}{5} \Rightarrow V_x = \frac{14}{5}\,\text{V}$$

Problem 3.13

Determine the mesh currents.

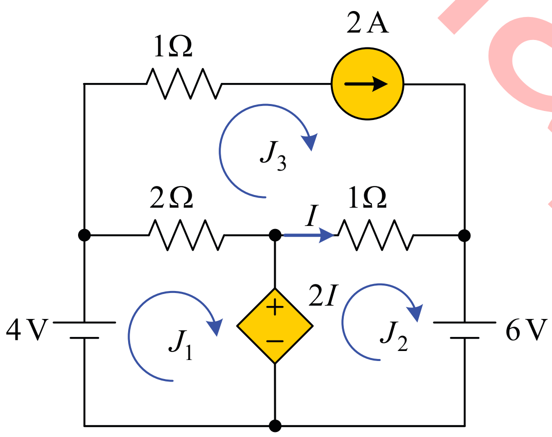

Figure 1

Solution

From the current source it is obvious that $J_3 = 2\,\mathrm{A}$. Also

$$I = J_2 - J_3$$

$$\Rightarrow I = J_2 - 2$$

The KVL in loops 1 & 2 gives the following equations

$$2J_1 - 2J_3 = 4 - 2I$$

$$I = 2I - 6$$

or

$$J_1 + J_2 = 6$$

$$I = 6 \Rightarrow J_2 - 2 = 6 \Rightarrow J_2 = 8\,\mathrm{A}$$

Therefore

$$J_1 + 8 = 6 \Rightarrow J_1 = -2\,\mathrm{A}$$

Problem 3.14

In the circuit of Fig.1 determine:

(a) $A_i = \dfrac{I_c}{I_b}$ (b) $A_V = \dfrac{V_{ce}}{V_{be}}$

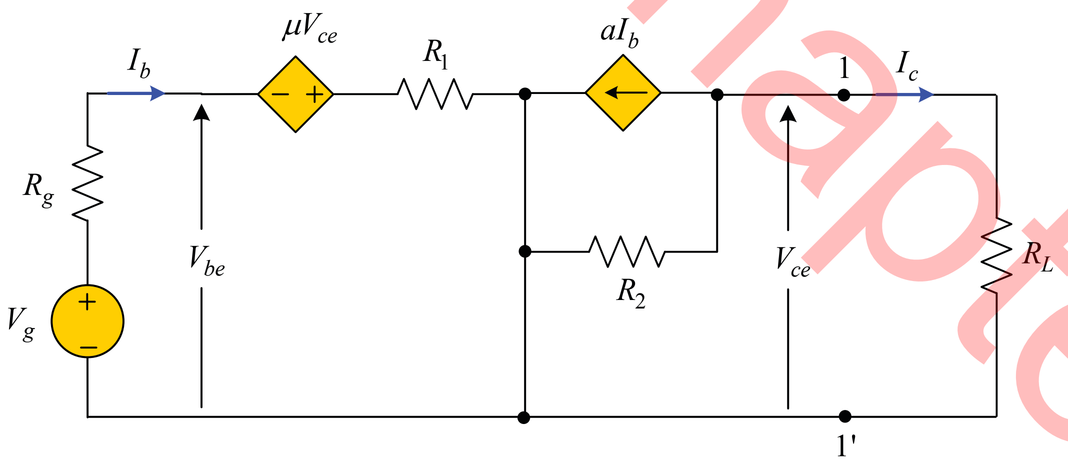

Figure 1

Solution

(a)We define the loop currents as in Fig. 2.

From the first loop we have $J_1 = I_b$, and

$$\left(R_g + R_1\right) J_1 = V_g + \mu V_{ce}$$

$$\Rightarrow J_1 = I_b = \frac{V_g + \mu V_{ce}}{R_g + R_1}$$

From the second loop we obtain

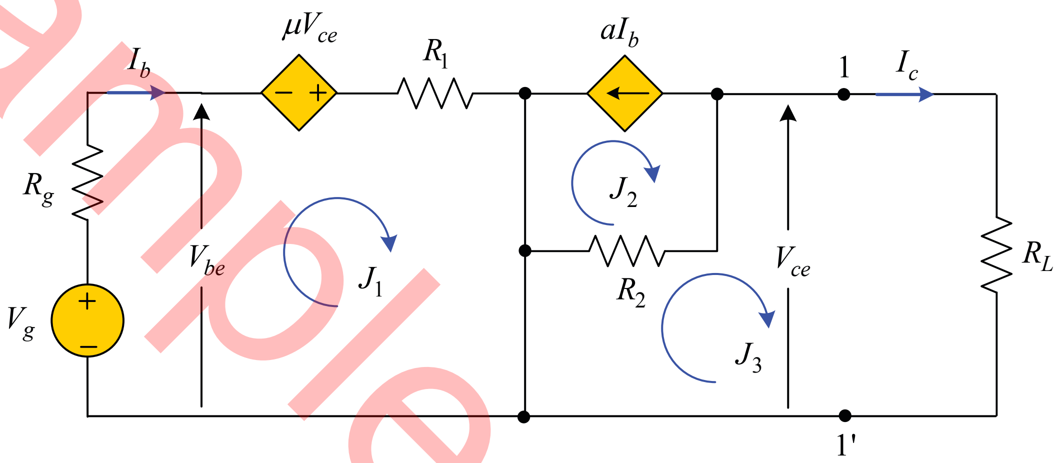

Figure 2

$$J_2 = -aI_b$$

$$\Rightarrow J_2 = -a\frac{V_g + \mu V_{ce}}{R_g + R_1}$$

From the third loop we have $J_3 = I_c$, and

$$-R_2 J_2 + (R_2 + R_L) J_3 = 0$$

$$\Rightarrow J_3 = I_c = \frac{R_2}{R_2 + R_L} J_2$$

$$\Rightarrow I_c = -a\frac{R_2}{R_2 + R_L}\frac{V_g + \mu V_{ce}}{R_g + R_1}$$

If we divide I_c and I_b we obtain

$$A_i = \frac{I_c}{I_b} = -a\frac{R_2}{R_2 + R_L}$$

(b) To obtain $A_V = \dfrac{V_{ce}}{V_{be}}$ we work as follows

$$A_V = \frac{V_{ce}}{V_{be}} = \frac{V_{ce}}{R_1 I_b - \mu V_{ce}}$$

$$= \frac{I_c R_L}{R_1 I_b - \mu I_c R_L} = \frac{A_i R_L}{R_1 - \mu R_L A_i}$$

Thus,

$$\Rightarrow A_V = \frac{-a\dfrac{R_2}{R_2 + R_L}R_L}{R_1 - \mu R_L - a\dfrac{R_2}{R_2 + R_L}} = \frac{-aR_2 R_L}{(R_2 + R_L)(R_1 - \mu R_L) - aR_2}$$

Problem 3.15

Find R_1 and R_2 so that $I_1 = -0.5\,\text{A}$ and $I_2 = -2.3\,\text{A}$.

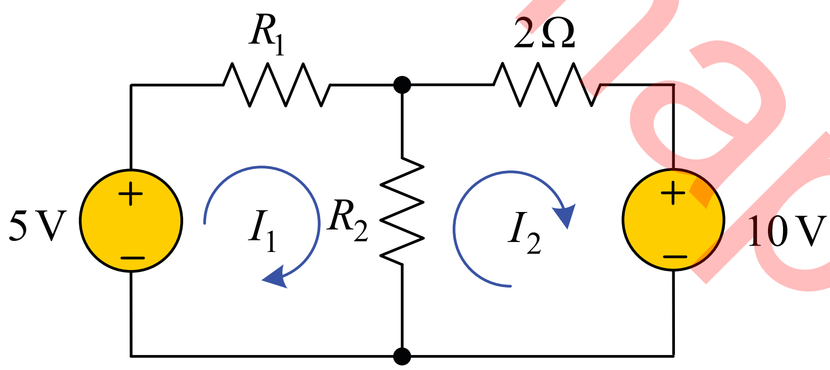

Figure 1

Solution

We have the following loop equations

$$I_1 R_1 + (I_1 - I_2)R_2 = 5$$

$$(I_2 - I_1)R_2 + 2I_2 = -10$$

If we replace the values of the currents we get

$$-0.5R_1 + 1.8R_2 = 5$$

$$-1.8R_2 - 4.6 = -10$$

The solution of this system gives

$$R_1 = 0.8\Omega$$

and

$$R_2 = 3\Omega$$

Problem 3.16

Find R_1 and R_2 so that $V_1 = 1\text{V}$ and $V_2 = 2\text{V}$.

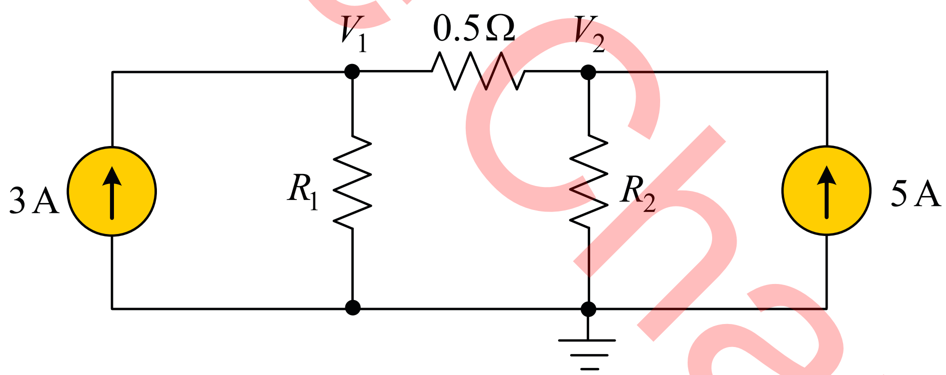

Figure 1

Solution

The nodal equations can be written as

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{0.5} = 3$$

$$\frac{V_2}{R_2} + \frac{V_2 - V_1}{0.5} = 5$$

The solution of this system regarding R_1 and R_2, gives

$$\frac{V_1}{R_1} = 3 - \frac{V_1 - V_2}{0.5}$$

$$\Rightarrow R_1 = \frac{V_1}{3 - \frac{V_1 - V_2}{0.5}} = 0.2\Omega$$

$$\frac{V_2}{R_2} = 5 - \frac{V_2 - V_1}{0.5}$$

$$\Rightarrow R_2 = \frac{V_2}{5 - \frac{V_2 - V_1}{0.5}} = \frac{2}{3}\Omega$$

Problem 3.17

Find I_1 and I_2.

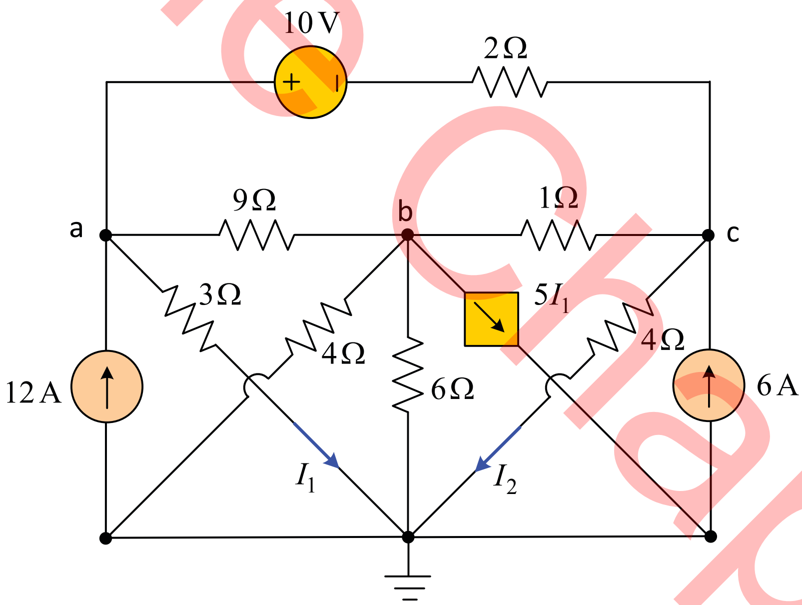

Figure 1

Solution

The circuit, except of the reference node, has three other nodes. We will solve the problem following the nodal method using matrices. The first step is the determination of the branch conductance matrix *G*. In this matrix the branch conductances G_{mm} are located on the main diagonal and the transconductances G_{mn} are located on the intersection of *m*th row and *n*th column.

Thus, the branch conductance matrix is equal to

$$\boldsymbol{G} = \begin{bmatrix} \frac{1}{3}+\frac{1}{9}+\frac{1}{2} & -\frac{1}{9} & -\frac{1}{2} \\ -\frac{1}{9} & \frac{1}{9}+\frac{1}{4}+\frac{1}{6}+1 & -1 \\ -\frac{1}{2} & -1 & 1+\frac{1}{2}+\frac{1}{4} \end{bmatrix}$$

The current matrix is

$$\boldsymbol{I} = \begin{bmatrix} 12+\frac{10}{2} \\ -5I_1 \\ 6-\frac{10}{2} \end{bmatrix} = \begin{bmatrix} 17 \\ -5I_1 \\ 1 \end{bmatrix}$$

Therefore, we have the following system of linear equations

$$\begin{bmatrix} \frac{1}{3}+\frac{1}{9}+\frac{1}{2} & -\frac{1}{9} & -\frac{1}{2} \\ -\frac{1}{9} & \frac{1}{9}+\frac{1}{4}+\frac{1}{6}+1 & -1 \\ -\frac{1}{2} & -1 & 1+\frac{1}{2}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 17 \\ -5I_1 \\ 1 \end{bmatrix}$$

The voltage V_a is equal to

$$V_a = 3I_1 \Rightarrow -5I_1 = -5\frac{V_a}{3}$$

Substituting I_1 the system becomes

$$\begin{bmatrix} \frac{1}{3}+\frac{1}{9}+\frac{1}{2} & -\frac{1}{9} & -\frac{1}{2} \\ \frac{5}{3}-\frac{1}{9} & \frac{1}{9}+\frac{1}{4}+\frac{1}{6}+1 & -1 \\ -\frac{1}{2} & -1 & 1+\frac{1}{2}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 17 \\ 0 \\ 1 \end{bmatrix}$$

Solving this system we get

$$V_a = 13.19\,\text{V}$$

$$V_b = -16.916\,\text{V}$$

$$V_c = -5.326\,\text{V}$$

Therefore

$$I_1 = \frac{V_a}{3} = 4.397\,\text{A}$$

$$I_2 = \frac{V_c}{4} = -1.332\,\text{A}$$

Problem 3.18

Find I_o in the circuit in Fig. 1.

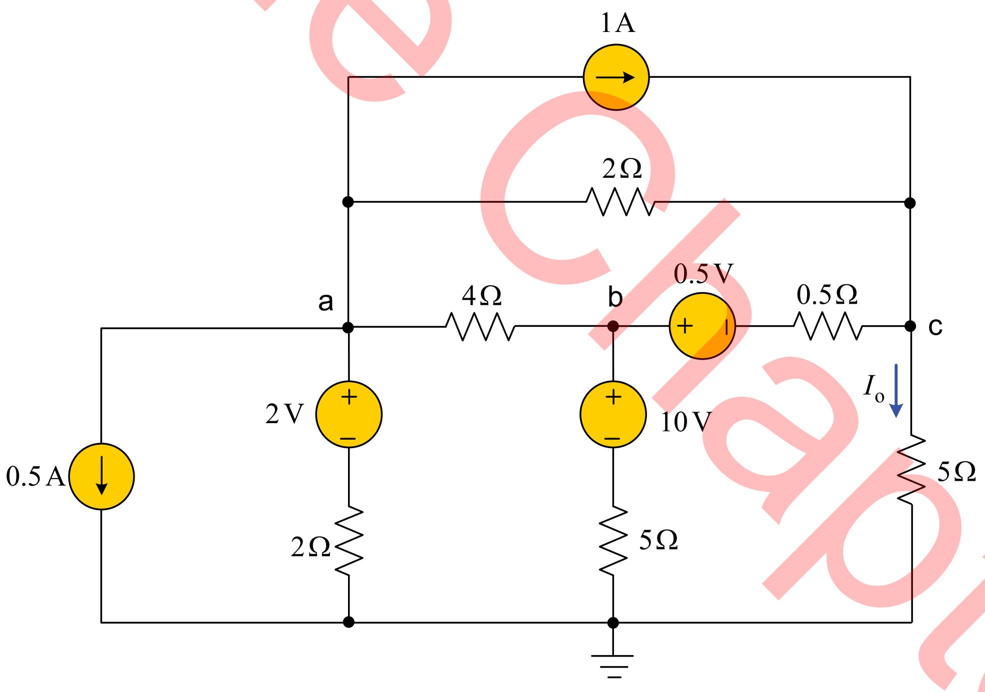

Figure 1

Solution

This circuit has three nodes except of the reference node. As in the previous problem, we will solve the problem following the nodal method using matrices. The branch conductance matrix is equal to

$$G = \begin{bmatrix} \frac{1}{2}+\frac{1}{4}+\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{0.5}+\frac{1}{4} & -\frac{1}{0.5} \\ -\frac{1}{2} & -\frac{1}{0.5} & \frac{1}{0.5}+\frac{1}{2}+\frac{1}{5} \end{bmatrix}$$

The current matrix is equal to

$$I = \begin{bmatrix} -0.5+\frac{2}{2}-1 \\ \frac{10}{5}+\frac{0.5}{0.5} \\ 1-\frac{0.5}{0.5} \end{bmatrix} = \begin{bmatrix} -0.5 \\ 3 \\ 0 \end{bmatrix}$$

Therefore we have the following linear system of equations

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4}+\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{5}+2+\frac{1}{4} & -2 \\ -\frac{1}{2} & -2 & 2+\frac{1}{2}+\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} -0.5 \\ 3 \\ 0 \end{bmatrix}$$

The solution of the above system gives

$$V_a = 1.87\,\mathrm{V}$$

$$V_b = 4.296\,\mathrm{V}$$

$$V_c = 3.528\,\mathrm{V}$$

Therefore

$$I_o = \frac{V_c}{5} = 0.705\,\mathrm{A}$$

Problem 3.19

Find V_{yx} in the circuit in Fig.1 using mesh analysis.

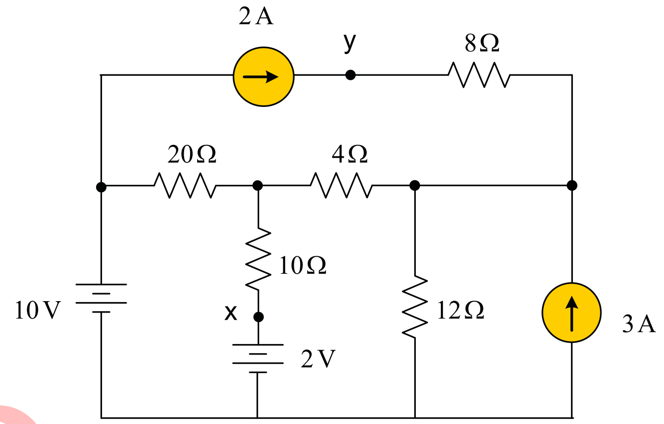

Figure 1

Solution

In the circuit of Fig. 2 we observe that $I_3 = 2\,\text{A}$ and $I_4 = -3\,\text{A}$. The mesh equations for meshes 1 and 2, respectively, are

$$30J_1 - 10J_2 - 20(2) = 10 - 2$$

$$-10J_1 + 26J_2 - 4(2) - 12(-3) = 2$$

or

$$30J_1 - 10J_2 = 48$$

$$-10J_1 + 26J_2 = -26$$

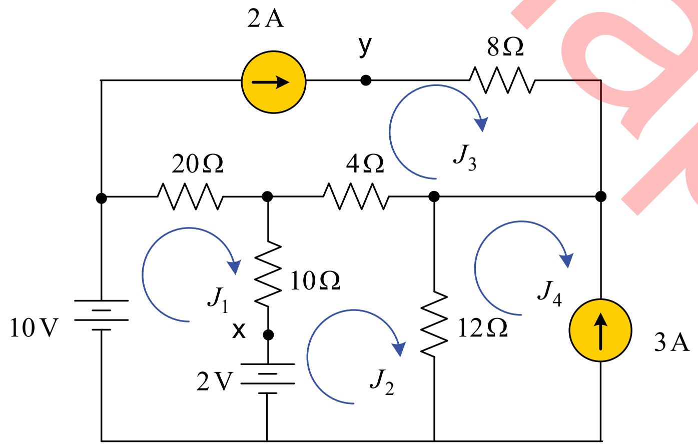

Figure 2

The solution of the above system is

$$J_1 = 1.453\,\text{A}$$

$$J_2 = -0.441\,\text{A}$$

Finally, the voltage V_{yx} is calculated as

$$V_{yx} = 8\times 2 + 4(2 - J_2) + 10(J_1 - J_2) = 44.706\,\text{V}$$

Problem 3.20

Determine the voltages at the nodes for the circuit in Fig. 1.

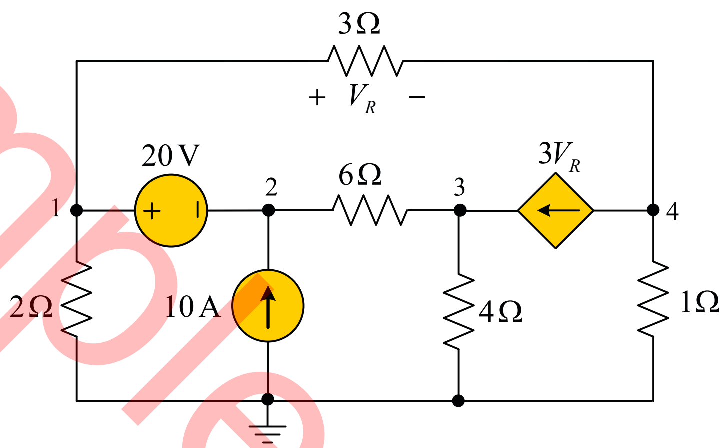

Figure 1

Solution

As it is shown in Fig. 2, nodes 1 and 2 and nodes 3 and 4 as well, define supernodes. The equations for the two supernodes can be written as

$$I_1 + I_2 = 10 + I_3 \qquad (1)$$

$$I_1 = I_3 + I_4 + I_5 \qquad (2)$$

The branch currents are then substituted in terms of the circuit node voltages:

$$\frac{V_1 - V_4}{3} + \frac{V_1}{2} = 10 + \frac{V_3 - V_2}{6} \Rightarrow 5V_1 + V_2 - V_3 - 2V_4 = 60 \qquad (3)$$

$$\frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} \Rightarrow 4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \qquad (4)$$

We need two additional equations. From the supernode we can observe that

$$V_1 - V_2 = 20 \qquad (5)$$

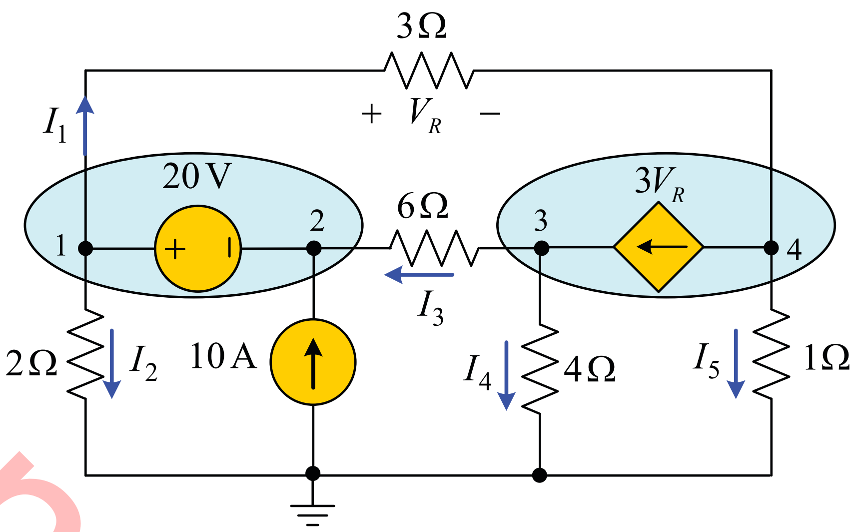

Figure 2

At node 4 we also have the following equation

$$I_1 = 3V_R + I_5 \Rightarrow \frac{V_1 - V_4}{3} = 3(V_1 - V_4) + V_4 \qquad (6)$$

$$\Rightarrow 8V_1 - 5V_4 = 0 \qquad (7)$$

The solution of the system of equations (3), (4), (5) and (7) gives

$$V_1 = 10.714\,\text{V},\ V_2 = -9.286\,\text{V},\ V_3 = -50\,\text{V} \text{ and } V_4 = 17.143\,\text{V}$$

Problem 3.21

Find V_{ab} in the circuit in Fig. 1.

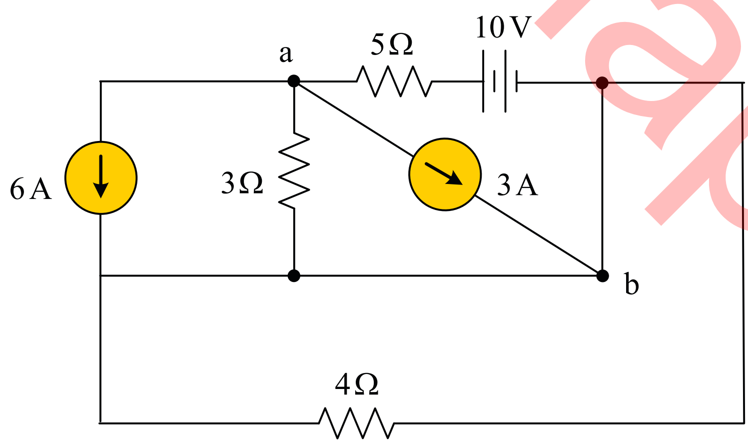

Figure 1

Solution

In the circuit of Fig. 2, the equation at node a has the form

$$\frac{V_a}{3}+6+3+\frac{V_a-10}{5}=0$$

Thus,

$$V_a=V_{ab}=-13.125 \text{ Volts}$$

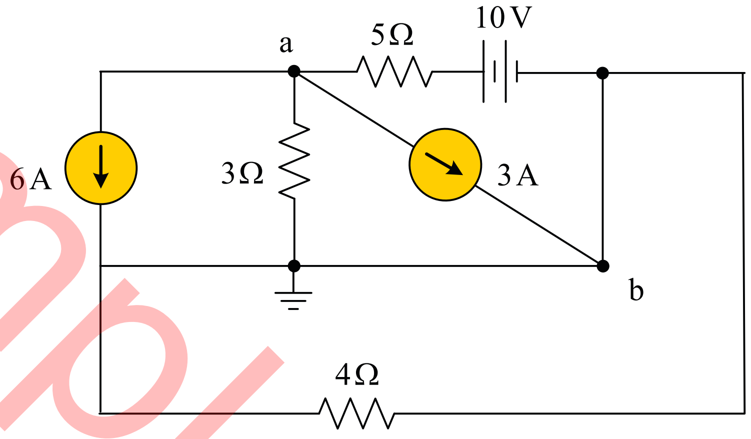

Figure 2

Problem 3.22

Find V_{ab} in the circuit in Fig. 1.

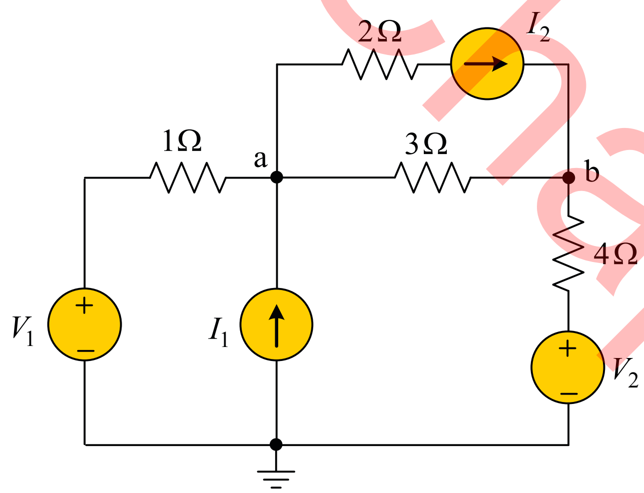

Figure 1

Solution

(a) Nodal technique.

The nodal equations are

$$\frac{V_a - V_1}{1} + \frac{V_a - V_b}{3} + I_2 = I_1 \Rightarrow \frac{4}{3}V_a - \frac{1}{3}V_b = I_1 - I_2 + V_1$$

$$\frac{V_a - V_b}{3} + I_2 = \frac{V_b - V_2}{4} \Rightarrow \frac{1}{3}V_a - \frac{7}{12}V_b = -I_2 - \frac{V_2}{4}$$

From this system we obtain

$$V_a = \frac{7I_1 - 3I_2 + 7V_1 + V_2}{8}$$

$$V_b = \frac{I_1 + 3I_2 + V_1 + V_2}{2}$$

Therefore

$$V_{ab} = \frac{3I_1 - 15I_2 + 3V_1 - 3V_2}{8}$$

(b) Mesh technique.

As we can see in Fig. 2, we suitably choose the loops in order to have the obvious equations $J_1 = I_1$ and $J_3 = I_2$. Thus, we need to extract only the equation in the second loop:

$$(1+3+4)J_2 - J_1 - 3J_3 = V_1 - V_2 \Rightarrow J_2 = \frac{I_1 + 3I_2 + V_1 - V_2}{8}$$

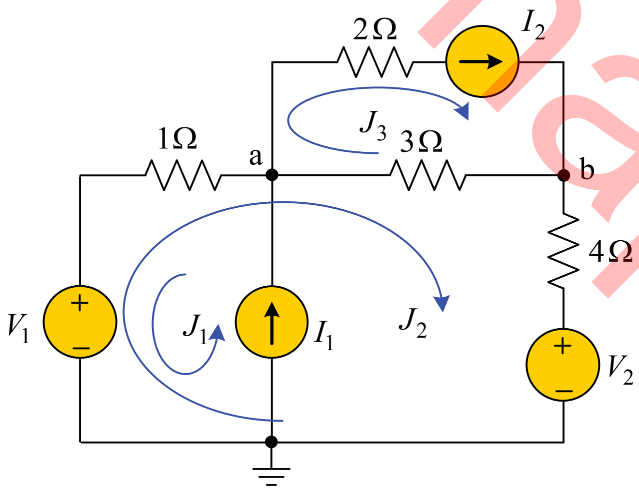

Figure 2

Finally

$$V_{ab} = 3(J_2 - J_3) = \frac{3I_1 - 15I_2 + 3V_1 - 3V_2}{8}$$

Problem 3.23

In the circuit of Fig. 1

(a) Obtain the voltages V_a and V_b.

(b) Calculate the power in the current source of 9A and find if this source absorbs or delivers power.

(c) Calculate the power in all sources and resistors and confirm the power balance.

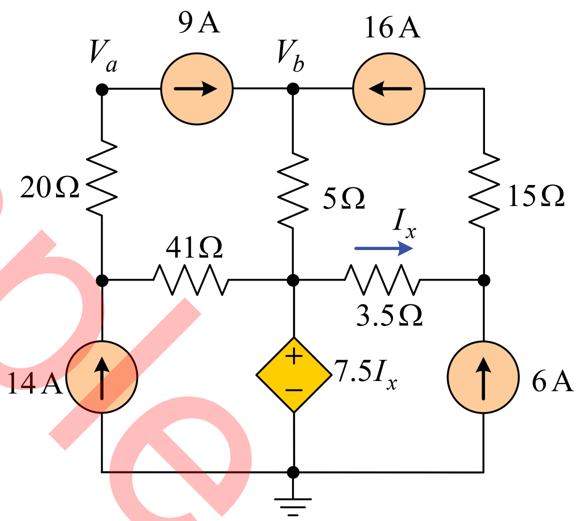

Figure 1

Solution

(a) Although we want the node voltages it is better to solve the problem using mesh analysis. This is because we can easily obtain the loop currents from the current sources. Specifically, for the circuit in Fig. 2 we conclude that

$$J_1 = 14\,\text{A},\ J_2 = 9\,\text{A},\ J_3 = -16\,\text{A} \text{ and } J_4 = -6\,\text{A}$$

Current I_x is equal to

$$I_x = J_4 - J_3 = -6 - (-16) = 10\,\text{A}$$

The voltages V_a and V_b are

$$V_a = -20J_2 + 41(J_1 - J_2) + 7.5I_x = 100\,\text{V}$$

$$V_b = 5(J_2 - J_3) + 7.5I_x = 200\,\text{V}$$

(b) The power in each source is equal to

$$P_9 = V_{ab}I_{ab} = (V_a - V_b)9 = -900\,\text{W}$$

$$P_{16} = -(V_b - V_f)16 = -16\left[5(J_2 - J_3) + 3.5J_x - 15J_3\right] = -6400\,\text{W}$$

$$P_{14} = -14V_c = -14\left[41(J_1 - J_2) + 7.5(J_4 - J_3)\right] = -3920\,\text{W}$$

$$P_6 = -6V_e = -6\left(-3.5I_x + 7.5I_x\right) = -240\,\text{W}$$

$$P_{dep} = \left(J_1 - J_4\right)\left(7.5I_x\right) = 1500\,\text{W}$$

Thus, all independent sources deliver power to the circuit, whereas the dependent source absorbs power.

(c) The power absorbed in each resistance is equal to

$$P_{20} = 20\left(J_2\right)^2 = 1620\,\text{W}$$

$$P_5 = 5\left(J_2 - J_3\right)^2 = 3125\,\text{W}$$

$$P_{15} = 15\left(J_3\right)^2 = 3840\,\text{W}$$

$$P_{41} = 41\left(J_2 - J_1\right)^2 = 1025\,\text{W}$$

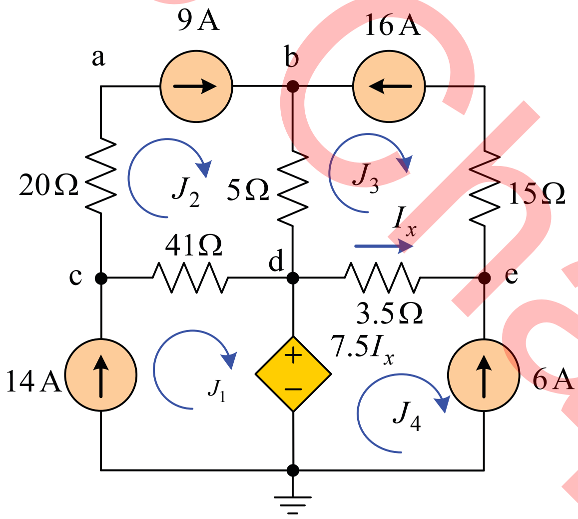

Figure 2

$$P_{3.5} = 3.5\left(J_x\right)^2 = 350\,\text{W}$$

From the above we conclude that

$$P_{delivered} = P_9 + P_{16} + P_{14} + P_6 = -11460\ \text{W}$$

$$P_{absorbed} = P_{3.5} + P_{20} + P_5 + P_{15} + P_{41} + P_{3.5} = 11460\ \text{W}$$

Problem 3.24

(a) Find the node voltages in the circuit in Fig. 1.

(b) Confirm the power balance.

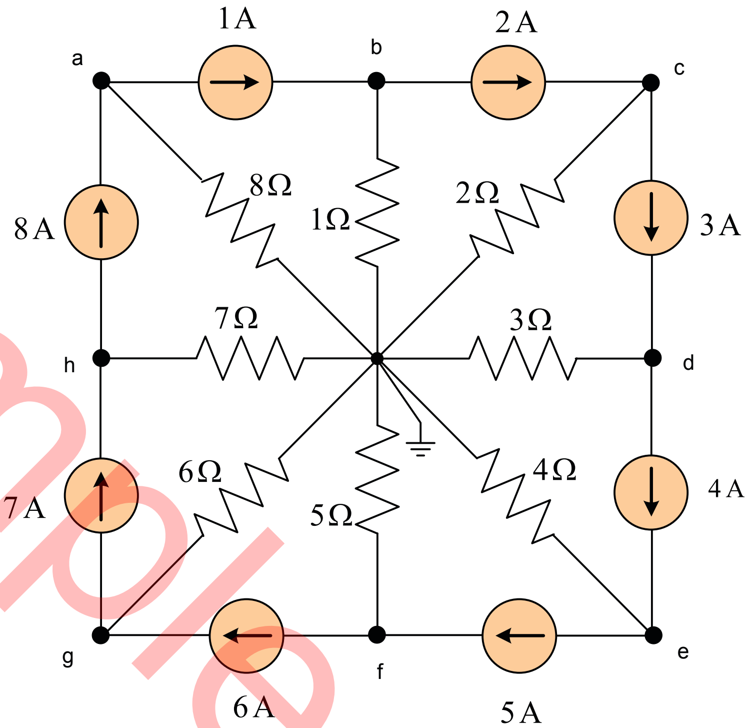

Figure 1

Solution

(a) In Fig. 2 we observe that

$$J_k = k\,\text{A},\ \ k=1,2,\ldots,8$$

Now, the node voltages are calculated as follows

$$V_a = 8(J_8 - J_1) = 8(8-1) = 56\,\text{V}$$

$$V_b = 1(J_1 - J_2) = (1-2) = -1\,\text{V}$$

$$V_c = 2(J_2 - J_3) = 2(2-3) = -2\,\text{V}$$

$$V_d = 3(J_3 - J_4) = 3(3-4) = -3\,\text{V}$$

$$V_e = 4(J_4 - J_5) = 4(4-5) = -4\,\text{V}$$

$$V_f = 5(J_5 - J_6) = 5(5-6) = -5\,\text{V}$$

$$V_g = 6(J_6 - J_7) = 6(6-7) = -6\,\text{V}$$

$$V_h = 7(J_7 - J_8) = 7(7-8) = -7\,\text{V}$$

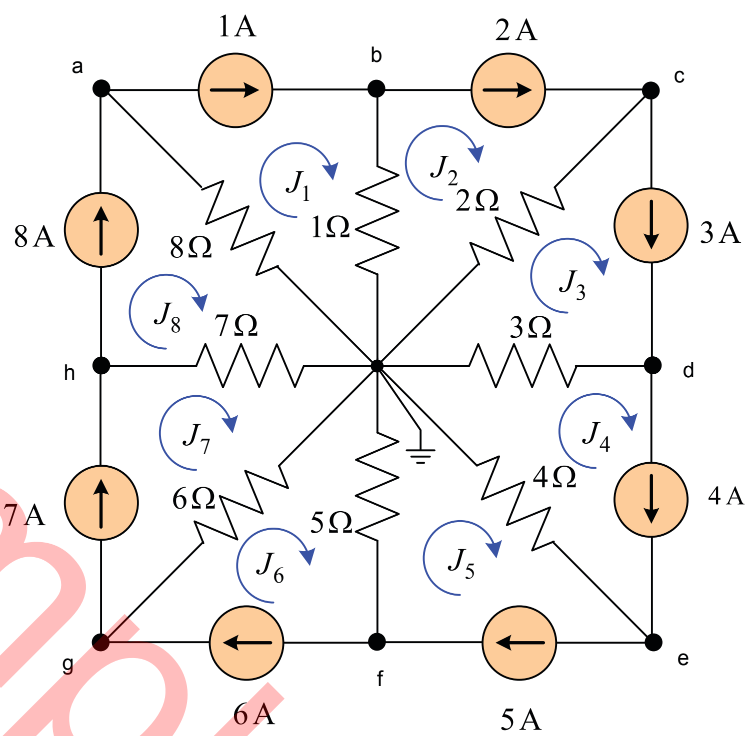

Figure 2

(b) The power in each current source is equal to

$$P_1 = 1(V_a - V_b) = 1(56+1) = 57\,\text{W}$$

$$P_2 = 2(V_b - V_c) = 2(-1+2) = 2\,\text{W}$$

$$P_3 = 3(V_c - V_d) = 3(-2+3) = 3\,\text{W}$$

$$P_4 = 4(V_d - V_e) = 4(-3+4) = 4\,\text{W}$$

$$P_5 = 5(V_e - V_f) = 5(-4+5) = 5\,\text{W}$$

$$P_6 = 6(V_f - V_g) = 6(-5+6) = 6\,\text{W}$$

$$P_7 = 7(V_g - V_h) = 7(-6+7) = 7\,\text{W}$$

$$P_8 = 8(V_h - V_a) = 8(-7-56) = -504\,\text{W}$$

From the above results it is clear that only the source of 8A delivers power to the circuit. The total power of the sources is equal to

$$P_s = \sum_{i=1}^{8} P_i = -420\,\text{W}$$

On the other hand, the total absorbed power by the resistors is

$$P_r = \sum_{i=1}^{8} R_i I_{R_i}^2 = 1\times1^2 + 2\times1^2 + \ldots + 7\times1^2 + 8\times7^2 = 420\,\mathrm{W}$$

Thus, we find that the delivered power is equal to the absorbed power.

Problem 3.25

For the circuit in Fig. 1 find the node voltages and the power of each source. What is the total power delivered by the sources to the circuit?

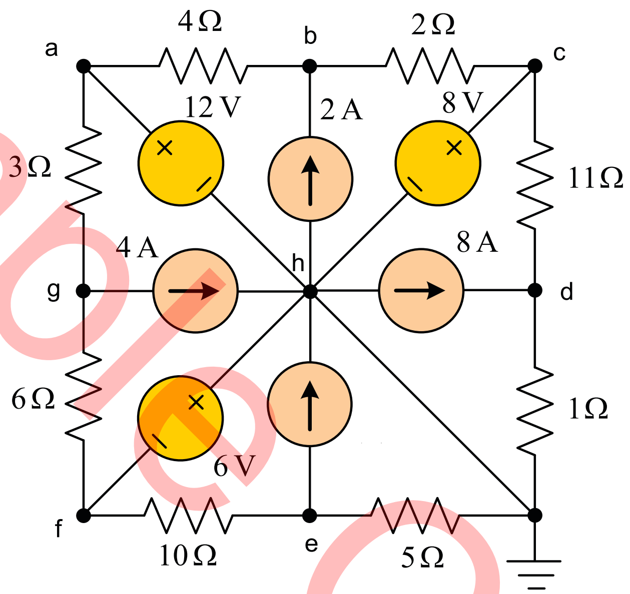

Figure 1

Solution

(b) Because of the short circuit, the voltage at node h is zero. Therefore, due to the voltage sources we have

$$V_a = 12\,\mathrm{V},\quad V_c = 8\,\mathrm{V} \text{ and } V_f = -6\,\mathrm{V}$$

The application of KCL in b, d, e and g nodes gives the following equations

node b: $$\frac{V_b - 12}{4} + \frac{V_b - 8}{2} = 2 \Rightarrow V_b = 12\,\mathrm{V}$$

node d: $$\frac{V_d}{1} + \frac{V_d - 8}{11} = 8 \Rightarrow V_d = 8\,\mathrm{V}$$

node e: $$\frac{V_e}{5} + \frac{V_e + 6}{10} = -3 \Rightarrow V_e = -12\,\mathrm{V}$$

node g: $\dfrac{V_g+6}{6}+\dfrac{V_g-12}{3}=-4 \Rightarrow V_g=-2\,\text{V}$

(b) In order to calculate the power at each one of the voltage sources we must determine the currents of the sources. We consider that the direction of the currents in the voltage sources is from + to - . Therefore

<u>Source 12 V</u>

$$I_{12}=\frac{V_g-12}{3}+\frac{V_b-12}{4}=\frac{-2-12}{3}+\frac{12-12}{4}=-\frac{14}{3}\text{A}$$

Power: $P_{V12}=12I_{12}=-56\,\text{W}$

<u>Source 8 V</u>

$$I_8=\frac{V_d-8}{11}+\frac{V_b-8}{2}=\frac{8-8}{11}+\frac{12-8}{2}=2\,\text{A}$$

Power: $P_{V8}=8I_8=16\,\text{W}$

<u>Source 6 V</u>

$$I_6=\frac{-6-V_e}{10}+\frac{-6-V_g}{6}=\frac{-6+12}{10}+\frac{-6+2}{6}=\frac{-1}{15}\text{A}$$

Power: $P_{V6}=6I_6=-\dfrac{6}{15}=-0.4\,\text{W}$

We know the voltages of the current sources. Therefore

<u>Source 2 A</u>

Power: $P_{I2}=2(-V_b)=2(-12)=-24\,\text{W}$

<u>Source 8 A</u>

Power: $P_{I8}=8(-V_d)=8(-8)=-64\,\text{W}$

<u>Source 3 A</u>

Power: $P_{I3}=3V_e=3(-12)=-36\,\text{W}$

<u>Source 4 A</u>

Power: $P_{I4}=4V_g=4(-2)=-8\,\text{W}$

Thus, the total delivered power by the sources is equal to

$$P_{s,\text{total}} = P_{V12} + P_{V8} + P_{V6} + P_{I2} + P_{I8} + P_{I3} + P_{I4}$$

$$= -56 + 16 - 0.4 - 24 - 64 - 36 - 8 = -172.4\,\text{W}$$

Problem 3.26

For the circuit in Fig. 1 determine V_o and I_o.

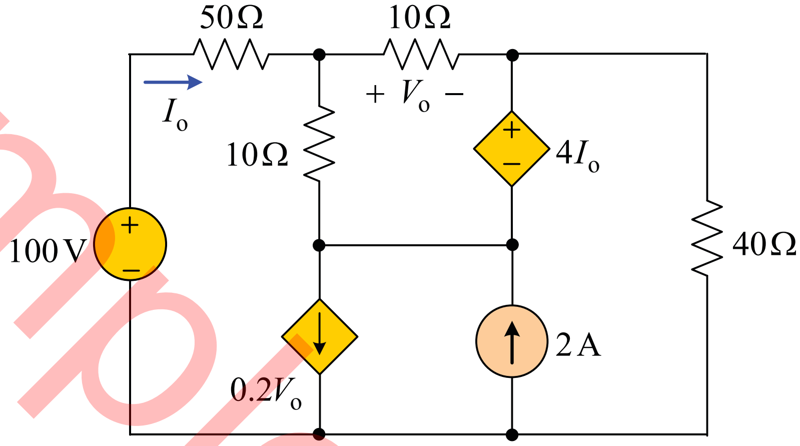

Figure 1

Solution

The two current sources are connected in parallel. Thus, they can be substituted with a current source of $2 - 0.2V_o$. The new equivalent circuit is depicted in Fig. 2. The application of the KVL in the three loops gives

$$J_1 = 0.2V_o - 2 \quad (1)$$

$$-10J_1 + 20J_2 - 10J_3 = -4I_o \quad (2)$$

$$60J_1 - 10J_2 + 100J_3 = 100 + 4I_o \quad (3)$$

Also

$$V_o = 10J_2 \quad (4)$$

and

$$I_o = J_1 + J_3 = 0.2V_o - 2 + J_3 \quad (5)$$

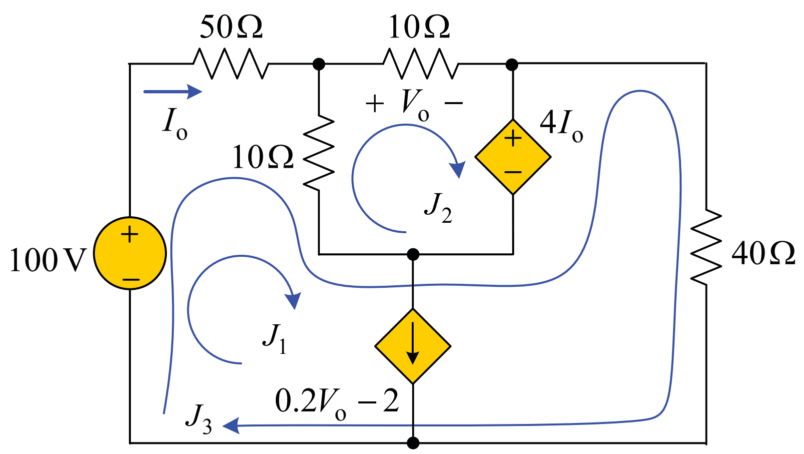

Figure 2

$$\Rightarrow I_o = 2J_2 + J_3 - 2 \tag{6}$$

Substituting eq. (4) into eq. (1) yields

$$J_1 = 2J_2 - 2 \tag{7}$$

Now, we substitute I_o and I_1 into equations (2) and (3)

$$-20J_2 + 20 + 20J_2 - 10J_3 = -8J_2 - 4J_3 + 8 \tag{8}$$

$$120J_2 - 120 - 10J_2 + 100J_3 = 100 + 8J_2 + 4J_3 - 8 \tag{9}$$

or

$$-8J_2 + 6J_3 = 12 \tag{10}$$

$$102J_2 + 96J_3 = 212 \tag{11}$$

The solution of the above system gives the following results

$$J_2 = 0.087\,\text{A} \text{ and } J_3 = 2.116\,\text{A} \tag{12}$$

Thus,

$$V_o = 10J_2 = 0.87\,\text{V} \tag{13}$$

$$I_o = 2J_2 + J_3 - 2 = 0.29\,\text{A} \tag{14}$$

Problem 3.27

For the circuit in Fig. 1 determine the node voltages.

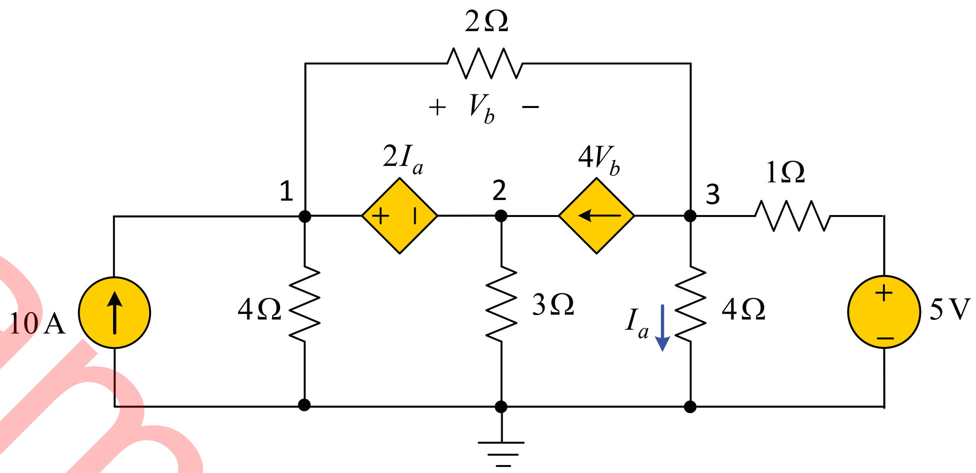

Figure 1

Solution

We can observe that there is a voltage source that connects two nodes. Therefore, as it is shown in Fig. 2, we consider a supernode that includes nodes 1 and 2.

We have the following nodal equations:

$$\text{supernode:} \quad \frac{V_1}{4} + \frac{V_1 - V_3}{2} + \frac{V_2}{3} = 10 + 4V_b \tag{1}$$

$$\text{node 3:} \quad \frac{V_3}{4} + \frac{V_3 - V_1}{2} + \frac{V_3 - 5}{1} + 4V_b = 0 \tag{2}$$

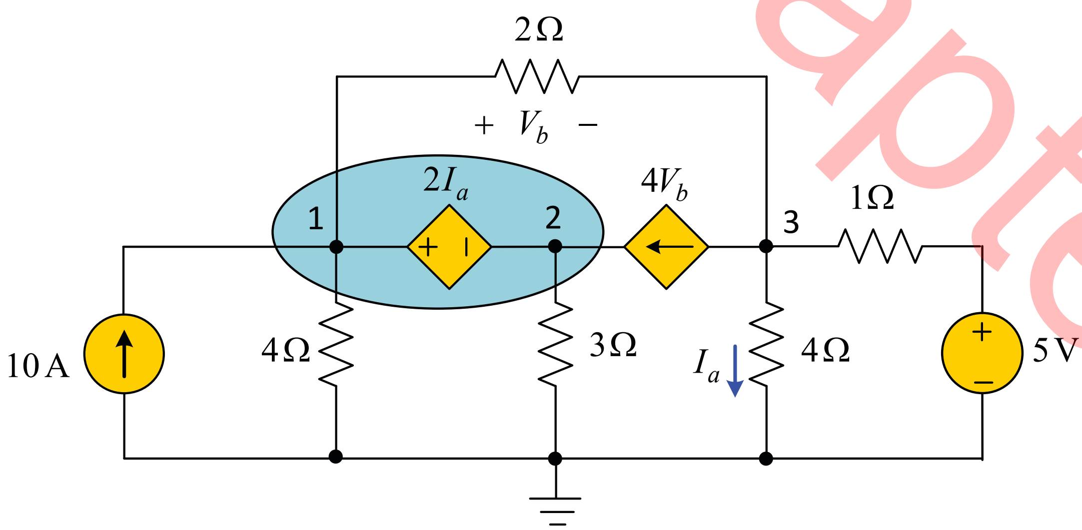

Figure 2

Also

$$I_a = \frac{V_3}{4} \quad (3)$$

$$V_b = V_1 - V_3 \quad (4)$$

$$V_1 - V_2 = 2I_a \Rightarrow V_1 - V_2 = 2\frac{V_3}{4} \Rightarrow 2V_1 - 2V_2 - V_3 = 0 \quad (5)$$

If we substitute V_b from equation (4) into equations (1) and (2) then we obtain the following system of linear equations

$$-39V_1 + 4V_2 + 42V_3 = 120 \quad (6)$$

$$14V_1 - 9V_3 = 20 \quad (7)$$

$$2V_1 - 2V_2 - V_3 = 0 \quad (8)$$

The solution of the above system gives

$$V_1 = 7.673\,\text{V},\ V_2 = 2.816\,\text{V} \text{ and } V_3 = 9.714\,\text{V}$$

Problem 3.28

For the circuit in Fig. 1 determine the node voltages.

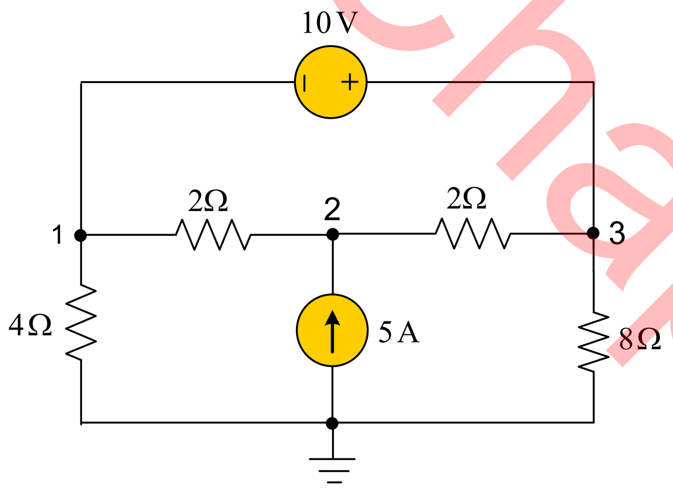

Figure 1

Solution

We observe that the source of 10V connects the nodes 1 and 3. Therefore, as we can see in Fig. 2, a supernode is defined that includes nodes 1 and 3.

The supernode equation is

$$I_3 = I_1 + I_4 + I_5 \qquad (1)$$

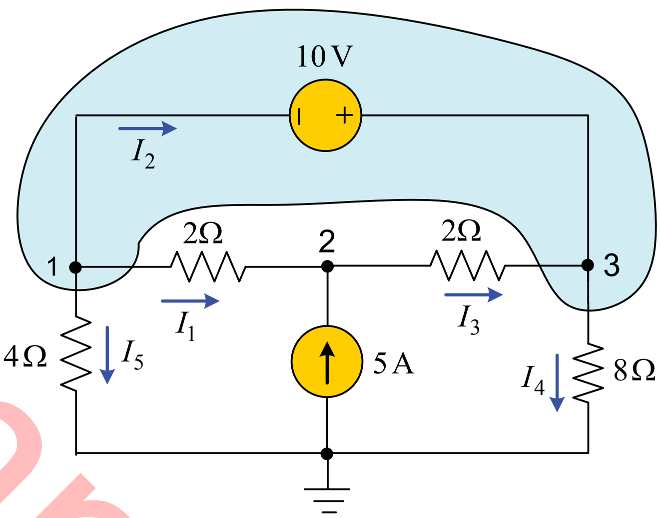

Figure 2

At node 2 we have the following equation

$$I_3 = I_1 + 5 \qquad (2)$$

At the supernode we also have the following obvious equation

$$V_1 + 10 = V_3 \qquad (3)$$

Substituting the currents in equations (1) and (2) with the node voltages we lead to the following system of linear equations:

$$\frac{V_2 - V_3}{2} = \frac{V_1 - V_2}{2} + \frac{V_3}{8} + \frac{V_1}{4} \Rightarrow 6V_1 - 8V_2 + 5V_3 = 0 \qquad (4)$$

$$\frac{V_2 - V_3}{2} = \frac{V_1 - V_2}{2} + 5 \Rightarrow -V_1 + 2V_2 - V_3 = 10 \qquad (5)$$

The solution of the system, consisting of the equations (3), (4) and (5), gives the node voltages:

$$V_1 = 10\text{V}, \quad V_2 = 20\text{V}, \text{ and } \quad V_3 = 20\text{V}$$

Problem 3.29

In the circuit of Fig. 1 determine I_1, I_2 and I_3 .

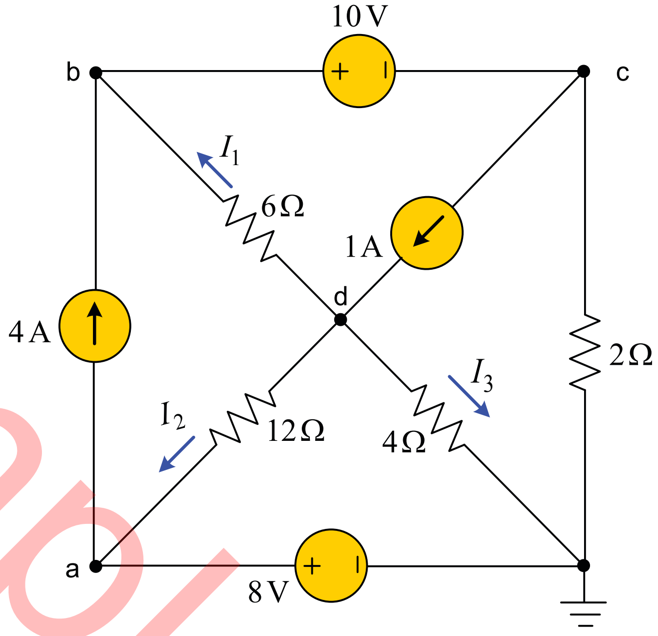

Figure 1

Solution

We observe that the circuit includes two current sources. Therefore, it is better to solve the problem by using mesh analysis.

As we can see in Fig. 2, there is a superloop. In the loop 3 it is clear that

$$J_3 = 4\text{A}$$

At branch c-d we have

$$J_1 - J_2 = 1\text{A}$$

The superloop equation is

$$6J_1 + 6J_2 - 6J_3 - 4J_4 = -10$$

Finally, the KVL in loop 4 yields

$$-4J_2 - 12J_3 + 16J_4 = 8$$

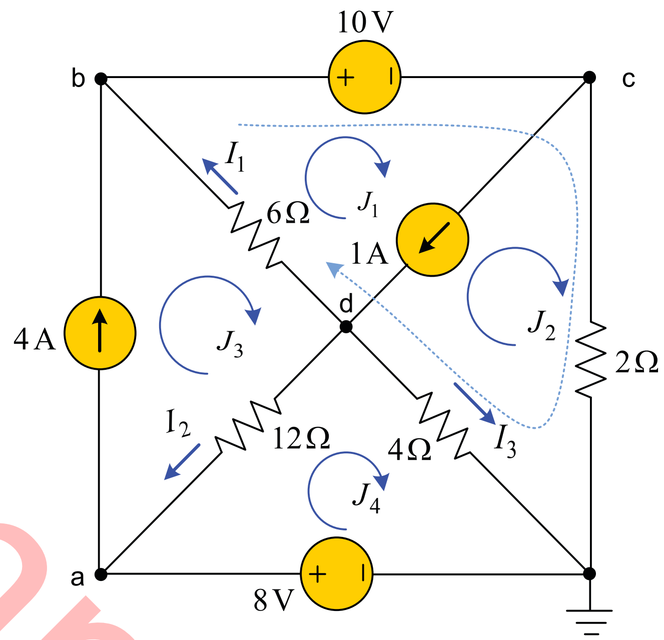

Figure 2

Substituting J_3 we get the following linear system

$$\begin{bmatrix} 1 & -1 & 0 \\ 3 & 3 & -2 \\ 0 & -2 & 8 \end{bmatrix}\begin{bmatrix} J_1 \\ J_2 \\ J_4 \end{bmatrix}=\begin{bmatrix} 1 \\ 7 \\ 28 \end{bmatrix}$$

whose solution is

$$J_1 = 3\,\text{A},\; J_2 = 2\,\text{A} \;\text{ and }\; J_4 = 4\,\text{A}$$

Therefore

$$I_1 = J_1 - J_3 = -1\,\text{A}$$

$$I_2 = J_3 - J_4 = 0\,\text{A}$$

$$I_3 = J_4 - J_2 = 2\,\text{A}$$

Problem 3.30

In the circuit of Fig. 1:

(a) Find the voltage V_o and the current I.

(b) What is the power in the current source?

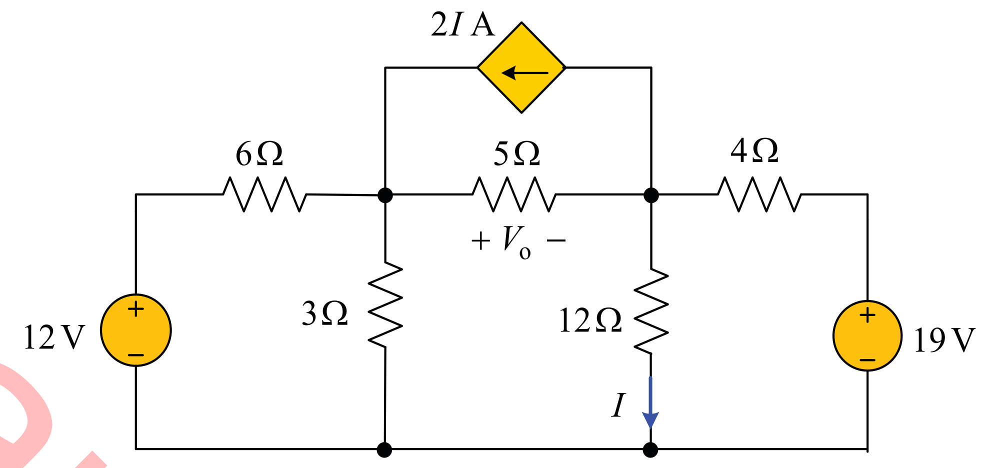

Figure 1

Solution

(a) We apply nodal analysis to the circuit in Fig. 2.
The node equations may be written as

node 1: $$\frac{V_1-12}{6}+\frac{V_1}{3}+\frac{V_1-V_2}{5}=2I$$

node 2: $$\frac{V_2}{12}+\frac{V_2-19}{4}+\frac{V_2-V_1}{5}=-2I$$

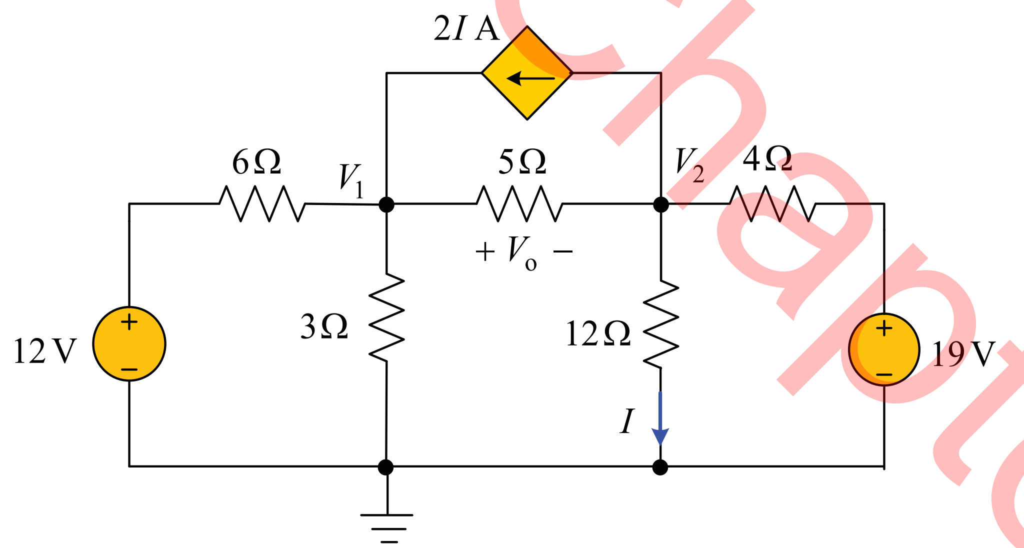

Figure 2

Also

$$V_2=12I$$

Substituting the current I we obtain

$$\frac{V_1-12}{6}+\frac{V_1}{3}+\frac{V_1-V_2}{5}=\frac{V_2}{6}\Rightarrow 21V_1-11V_2=60$$

$$\frac{V_2}{12}+\frac{V_2-19}{4}+\frac{V_2-V_1}{5}=-\frac{V_2}{6}\Rightarrow -6V_1+21V_2=142.5$$

The solution of the above system yields

$$V_1=7.54\,\text{V} \text{ and } V_2=8.94\,\text{V}$$

Therefore

$$I=\frac{V_2}{12}=0.745\,\text{A}$$

and

$$V_o=V_1-V_2=-1.4\,\text{V}$$

(b) The power of the current source is equal to

$$P=-V_o 2I=-(-1.4)\times 0.75=2.086\,\text{W}$$

Thus, the current source seems to absorb 2.086 W power.

Problem 3.31

Find the node voltages v_1, v_2, v_3, v_4 and v_5.

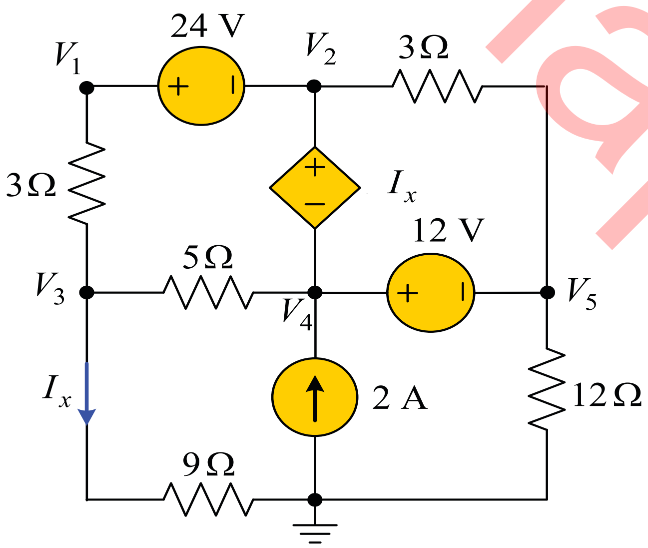

Figure 1

Solution

As it is shown in Fig. 2, we consider a supernode.
We have the following obvious equations:

$$V_4 - V_5 = 12\,\text{V}$$

$$V_2 - V_4 = 4I_x$$

Also

$$I_x = \frac{V_3}{9}$$

The equation at node 3 is

$$\frac{V_3}{9} + \frac{V_3 - V_4}{5} + \frac{V_3 - 24 - V_2}{3} = 0$$

Finally, the equation at supernode gives

$$\frac{V_3 - V_4}{5} + \frac{V_3 - 24 - V_2}{3} + 2 - \frac{V_5}{12} = 0$$

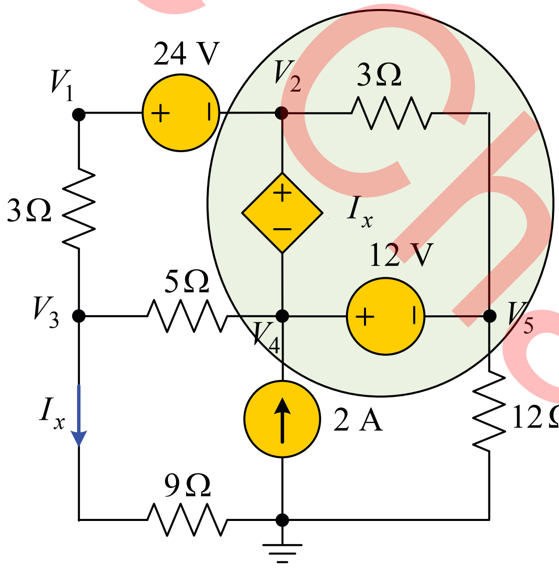

Figure 2

Eliminating the current I_x we will get the following linear system

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & -\frac{4}{9} & -1 & 0 \\ -\frac{1}{3} & \frac{1}{9}+\frac{1}{5}+\frac{1}{3} & -\frac{1}{5} & 0 \\ -\frac{1}{3} & \frac{1}{5}+\frac{1}{3} & -\frac{1}{5} & -\frac{1}{12} \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 8 \\ 6 \end{pmatrix}$$

The solution of this system is

$$V_2 = 15.975\,\text{V},\; V_3 = 22.528\,\text{V},\; V_4 = 5.963\,\text{V} \quad \text{and} \quad V_5 = -6.037\,\text{V}$$

Thus,

$$V_1 = 24 + V_2 = 39.975\,\text{V}$$

Problem 3.32

In the circuit of Fig. 1 find

(a) The node voltages

(b) The power in each one of the sources

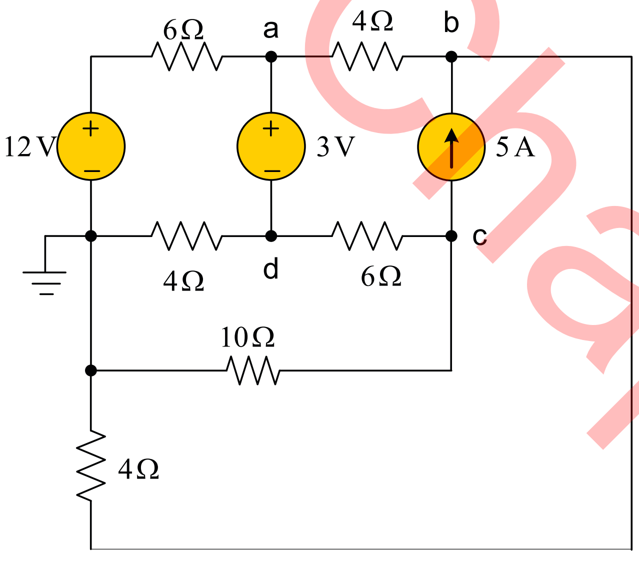

Figure 1

Solution

(a) We apply nodal analysis.

As it is shown in Fig. 2, we define a supernode that gives the following equations:

$$V_a - V_d = 3\,\text{V}$$

$$\frac{V_a - 12}{6} + \frac{V_a - V_b}{4} + \frac{V_d - V_c}{6} + \frac{V_d}{4} = 0$$

$$\Rightarrow 5V_a - 3V_b - 2V_c + 5V_d = 24$$

At nodes b and c we also have

$$\frac{V_b - V_a}{4} + \frac{V_b}{4} = 5 \Rightarrow -V_a + 2V_b = 20$$

$$\frac{V_c}{10} + \frac{V_c - V_d}{6} + 5 = 0$$

$$\Rightarrow -8V_c + 5V_d = 150$$

The solution of the above system of the four equations gives

$$V_a = 03.828\,\text{V},\ V_b = 11.914\,\text{V},\ V_c = -18.233\,\text{V}$$

and

$$V_d = 0.828\,\text{V}$$

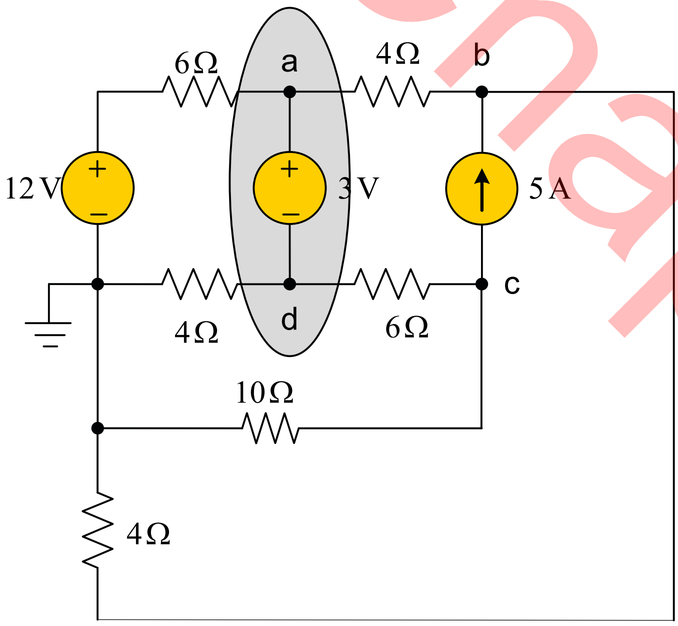

Figure 2

(b) The power in the sources can be found as the product of voltage and current. Specifically

$$P_{12} = 12\left(\frac{V_a - 12}{6}\right) = -16.435\,\text{W} \quad \text{(delivers power to the circuits)}$$

The current flowing through the 3V source is equal to the sum of currents that flowing through the 6Ω and 4Ω resistances. Therefore

$$P_3 = 3\left(\frac{12 - V_a}{6} + \frac{V_b - V_a}{4}\right) = 10.151\,\text{W} \quad \text{(absorbs power)}$$

$$P_5 = (V_b - V_c)(-5) = -152.802\,\text{W} \quad \text{(delivers power to the circuits)}$$

Problem 3.33

In the circuits of Fig. 1 find:

(a) I_1 and V_2

(b) The power in the current sources 10A and $3V_R$. Define if these sources deliver or absorb power.

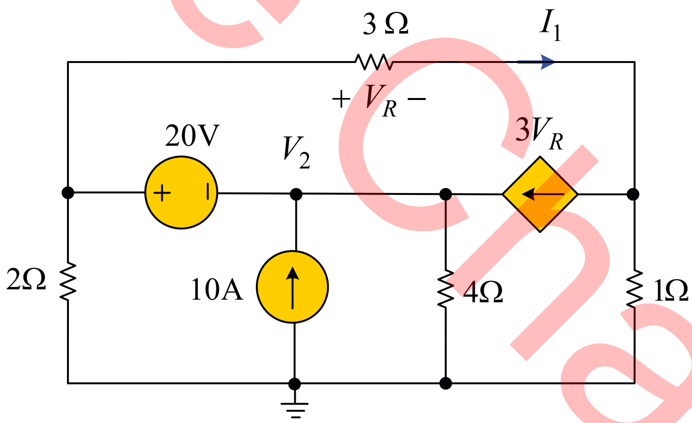

Figure 1

Solution

(a) Using Norton to Thevenin conversion, the circuit takes the equivalent form shown in Fig. 2. In order to solve the circuit, we will apply nodal analysis. As it is shown in the circuit of Fig. 2, a supernode is defined that includes nodes 1 and 2. In the supernode, we have the following obvious equation

$$V_1 - V_2 = 20$$

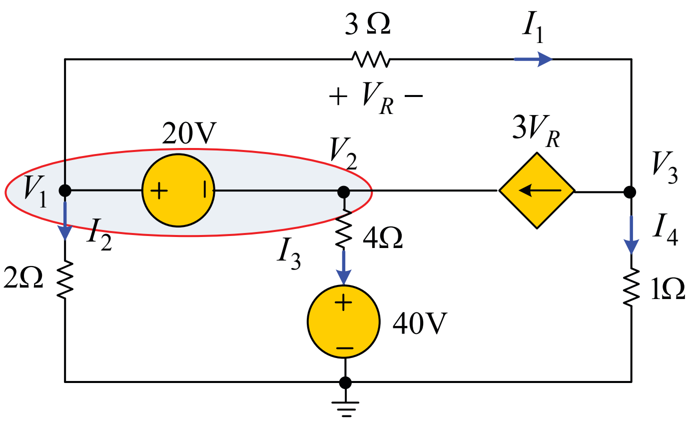

Figure 2

The node equations at the supernode and at node 3 are

$$3V_R = I_1 + I_2 + I_3 \Rightarrow 3(V_1 - V_3) = \frac{V_1 - V_3}{3} + \frac{V_1}{2} + \frac{V_2 - 40}{4}$$

$$I_1 = 3V_R + I_4 \Rightarrow \frac{V_1 - V_3}{3} = 3(V_1 - V_3) + \frac{V_3}{1}$$

The solution of the above system gives

$$V_1 = 6.383\text{V} \quad V_2 = -13.17\text{V} \;\text{ and }\; V_3 = 10.21\text{V}$$

Therefore

$$I_1 = \frac{V_1 - V_3}{3} = -1.2757\text{A}$$

(b) The current source of 10A absorbs power equal to

$$P_{10\text{A}} = V_2(-10) = (-13.17) \times (-10) = 136.17\text{W}$$

Finally, the depended source delivers the following power

$$P_{3V_R} = (V_2 - V_3)(-3V_R) = (V_2 - V_3)(-3(V_1 - V_3)) = -273.558\text{W}$$

Problem 3.34

Find I in the circuit in Fig. 1.

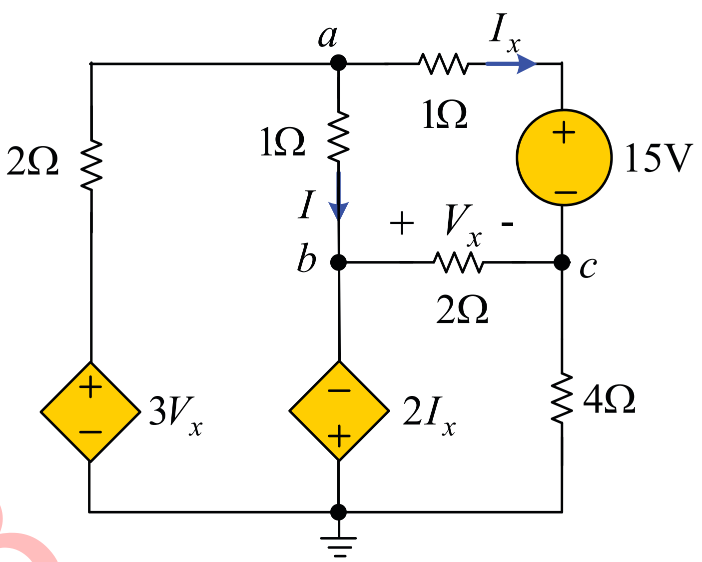

Figure 1

Solution

For pedagogical reasons, we will solve the circuit by applying both mesh and nodal analysis techniques.

Mesh analysis

In the circuit of Fig. 2 we have the following loop equations

$$3I_1 - I_2 = 2I_x + 3V_x \quad (1)$$

$$-I_1 + 4I_2 - 2I_3 = -15 \quad (2)$$

$$2I_2 + 6I_3 = 2I_x \quad (3)$$

Also

$$I_2 = I_x \quad (4)$$

$$V_x = 2(I_3 - I_2) \quad (5)$$

From equations (3) and (4) we have

$$2I_x + 6I_3 = 2I_x \Rightarrow I_3 = 0 \quad (6)$$

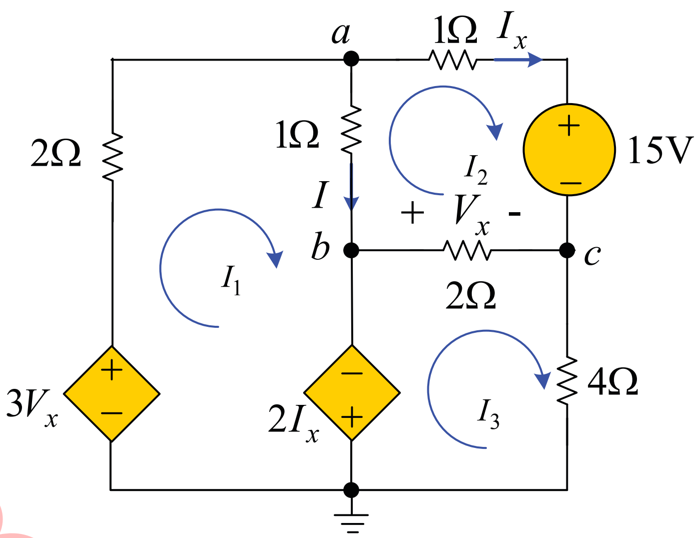

Figure 2

Therefore, from equations (1) and (2) we obtain

$$3I_1 - 3I_2 = 3(-2I_2) \Rightarrow I_1 = -I_2 \qquad (7)$$

$$-I_1 - 4I_1 = -15 \Rightarrow I_1 = 3\,\text{A} \qquad (8)$$

and

$$I = I_1 - I_2 = 2I_1 = 6\,\text{A}$$

<u>Nodal analysis</u>

From the original circuit we have the following nodal equations:

node a: $$\frac{V_a - V_b}{1} + \frac{V_a - 3V_x}{2} + \frac{V_a - 15 - V_c}{1} = 0 \Rightarrow 2.5V_a - V_b - V_c = 15 + 1.5V_x \qquad (9)$$

node b: $$V_b = -2I_x = -2(V_a - 15 - V_c) \Rightarrow 2V_a + V_b - 2V_c = 30 \qquad (10)$$

node c: $$V_a - 15 - V_c = \frac{V_c}{4} + \frac{V_c - V_b}{2} \Rightarrow 4V_a + 2V_b - 7V_c = 60 \qquad (11)$$

If we multiply eq. (10) by 2 and then subtract it from eq.(11) we will find that $V_c = 0\,\text{V}$. Since $V_x = V_b - V_c = V_b$ from eq. (9) we get

$$2.5V_a - 2.5V_b = 15 \Rightarrow I = V_a - V_b = \frac{15}{2.5} = 6\,\text{A}$$

Problem 3.35

Find I_1 in the circuit in Fig. 1.

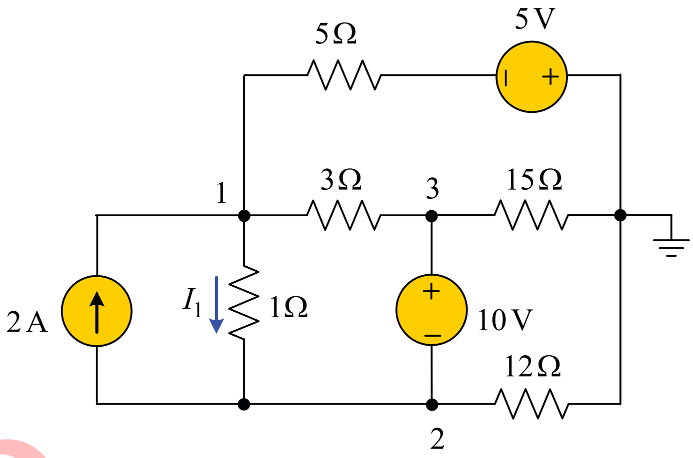

Figure 1

Solution

The circuit has four nodes. As shown in Figure 2, in order to apply the node method we consider a supernode consisting of nodes 2 and 3. Thus, we have the following nodal equations:

node 1: $I_1 + I_2 + I_3 = 2$

supernode: $I_2 + I_6 + I_5 + I_1 = 2$

where

$$I_1 = \frac{E_1 - E_2}{1}, \quad I_2 = \frac{E_1 - E_3}{3}, \quad I_3 = \frac{E_1 + 5}{5}$$

$$I_5 = \frac{-E_2}{12} \quad \text{and} \quad I_6 = \frac{-E_3}{15}$$

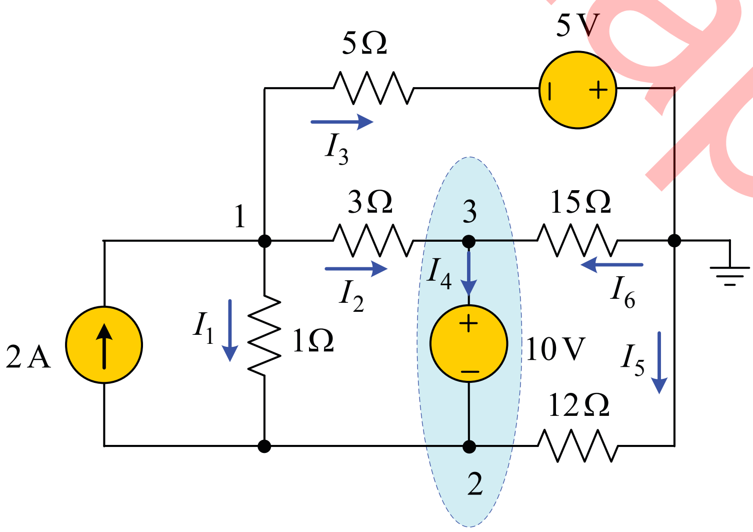

Figure 2

Substituting the above expressions to the nodal equations we get

$$\frac{E_1-E_2}{1}+\frac{E_1-E_3}{3}+\frac{E_1+5}{5}=2$$

$$\frac{E_1-E_3}{3}+\frac{-E_3}{15}+\frac{-E_2}{12}+\frac{E_1-E_2}{1}=2$$

or

$$23E_1-15E_2-5E_3=15$$

$$80E_1-65E_2-24E_3=120$$

But

$$E_3=E_2+10$$

Substituting E_3 we lead to the following system

$$23E_1-20E_2=65$$

$$80E_1-89E_2=360$$

The solution of the system is

$$E_1=-3.166\text{V} \quad \text{and} \quad E_2=-6.89\text{V}$$

Thus,

$$I_1=\frac{E_1-E_2}{1}=3.725\,\text{A}$$

Problem 3.36

Find I_x in the circuit in Fig. 1.

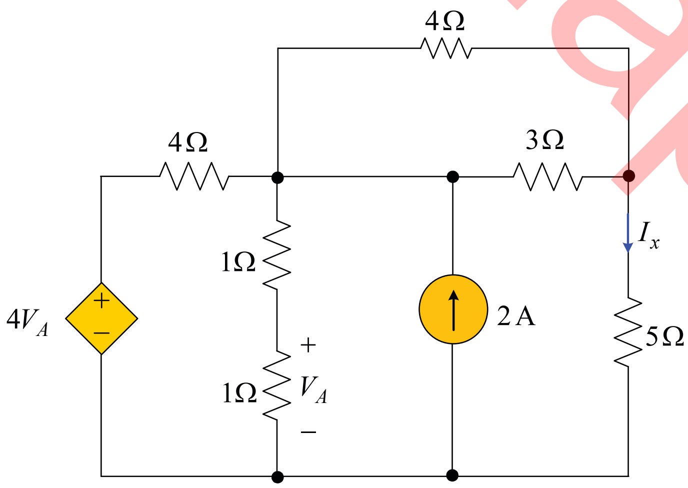

Figure 1

Solution

The circuit has four loops. We select the loops as shown in Fig. 2 in order to have the following obvious equation

$$J_4 = 2\text{A}$$

The application of the KVL in the other three loops gives the equations

$$6J_1 - 2J_2 = 4V_A$$

$$-2J_1 + 10J_2 - 3J_3 + 8J_4 = 0$$

$$-3J_2 + 7J_3 - 3J_4 = 0$$

In the depended source we also have the relation

$$V_A = J_1 - J_2$$

Substituting V_a we get the system

$$J_1 + J_2 = 0$$

$$-2J_1 + 10J_2 - 3J_3 = -16$$

$$-3J_2 + 7J_3 = 6$$

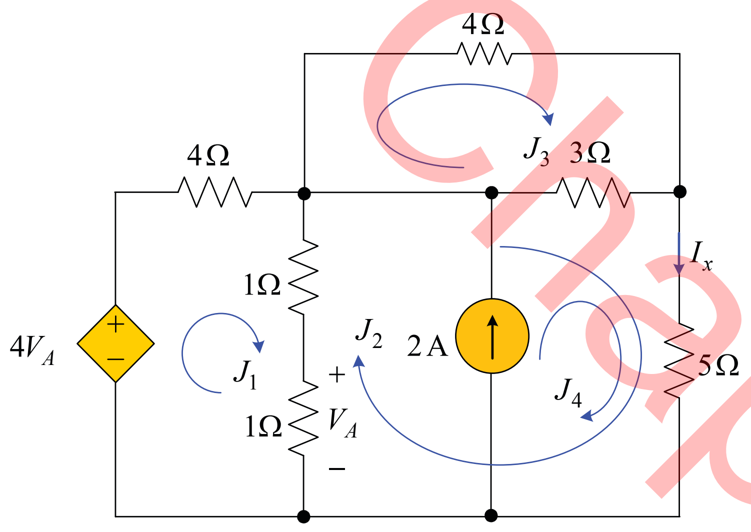

Figure 2

The solution of the system is

$$J_1 = 1.253\text{A}\,,\ \ J_2 = -1.253\text{A} \quad \text{and} \quad J_3 = 0.32\ \text{A}$$

Thus,

$$I_x = J_2 + J_4 = 0.747\,\text{A}$$

Problem 3.37

In the circuit of Fig. 1 find V_x and I_y.

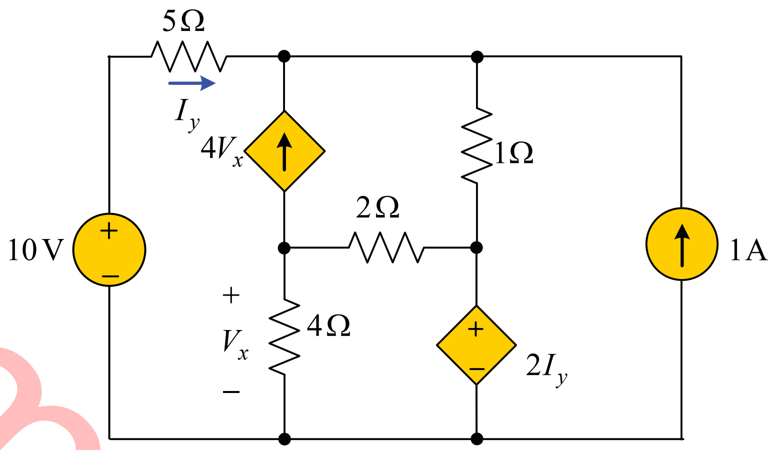

Figure 1

Solution

In order to take advantage of the two current sources and simplify the solution, we solve the circuit using mesh analysis. Thus, we select the loops as shown in Fig. 2. It is clear that

$$I_4 = -1\text{A} \;\text{ and }\; I_2 = 4V_x$$

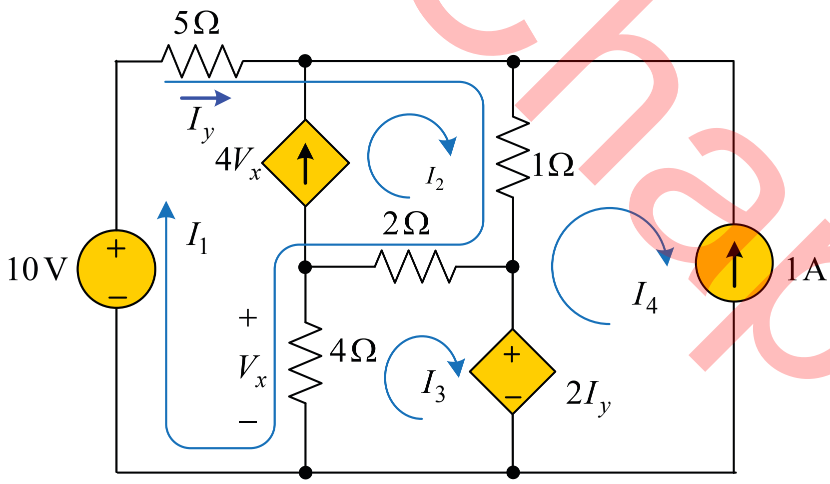

Figure 2

The equations in loops 1 and 3 are

$$12I_1 + 3I_2 - 6I_3 - I_4 = 10$$

$$-6I_1 - 2I_2 + 6I_3 = -2I_y$$

However

$$I_y = I_1$$

and

$$V_x = 4(I_1 - I_3) = 4I_y - 4I_3 \Rightarrow I_3 = I_y - \frac{1}{4}V_x$$

Thus, the loop equations become

$$12I_y + 12V_x - 6\left(I_y - \frac{1}{4}V_x\right) + 1 = 10$$

$$-6I_y - 8V_x + 6\left(I_y - \frac{1}{4}V_x\right) = -2I_y$$

or

$$6I_y + 13.5V_x = 9$$

$$2I_y - 9.5V_x = 0$$

Thus, the solution of the above system yields

$$I_y = 1.018\,\text{A} \;\; \text{and} \;\; V_x = 0.214\,\text{V}$$

Problem 3.38

In the circuit of Fig. 1 find the power in the voltage source of 4V.

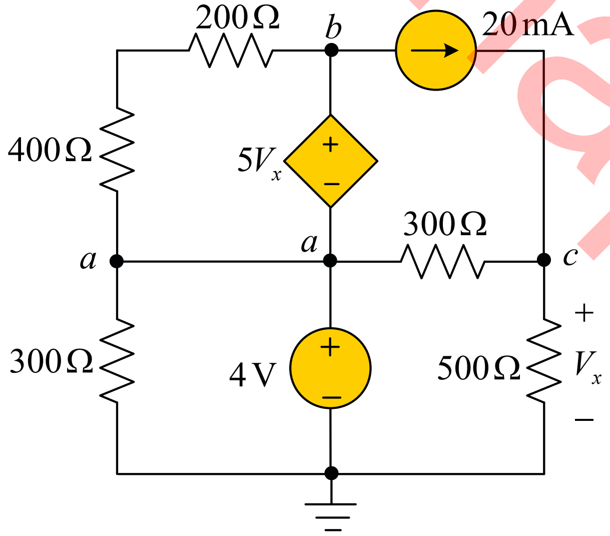

Figure 1

Solution

In the circuit in Fig. 2 we observe that we have the following obvious equations

$$V_a = 4\text{V}$$

$$\text{V}_\text{b} - \text{V}_\text{a} = 5\text{V}_\text{x} \Rightarrow \text{V}_\text{b} - 4 = 5\text{V}_\text{c}$$

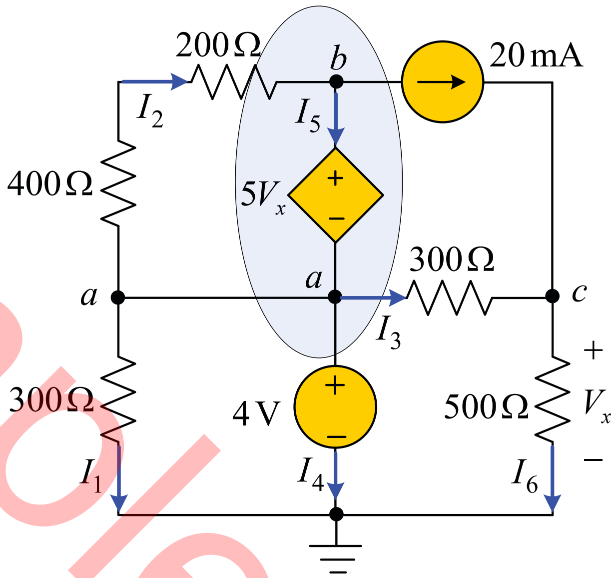

Figure 2

At node c, the node equation can be written as

$$\frac{V_c}{500} + \frac{V_c - 4}{300} = 20 \times 10^{-3}$$

From the above equation we get

$$V_c = 6.25\text{V}$$

Therefore

$$\text{V}_\text{b} = 4 + 5\text{V}_\text{c} = 35.25\text{V}$$

Now, we can determine I_4 and I_5:

$$I_4 = -I_1 - I_6 = -\frac{4}{300} - \frac{6.25}{500} = -25.833\,m\text{A}$$

$$\text{I}_5 = \frac{4 - V_b}{600} - 20\,m\text{A} = -72.083\,m\text{A}$$

Thus,

$$P_{5V_x} = (V_b - 4) I_5 = -2.253\,\text{W}$$

$$P_{4V} = 4I_4 = -0.103\,\text{W}$$

Problem 3.39

Use nodal analysis to find current I_o.

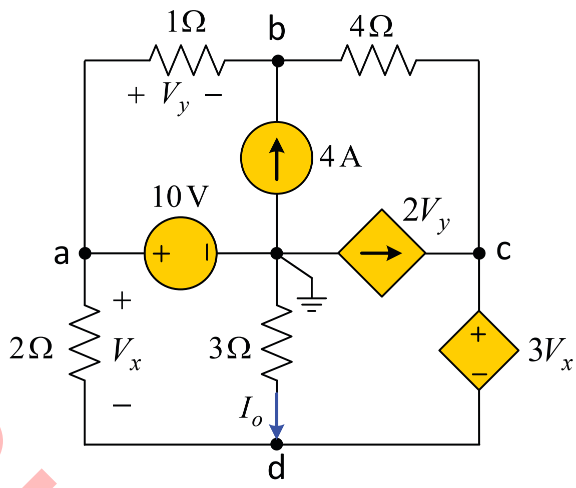

Figure 1

Solution

As we can observe in Fig. 2, the circuit has a supernode. The Supernode constraint equation is

$$V_c - V_d = 3V_x \quad (1)$$

At node a we also have

$$V_a = 10\,\text{V} \quad (2)$$

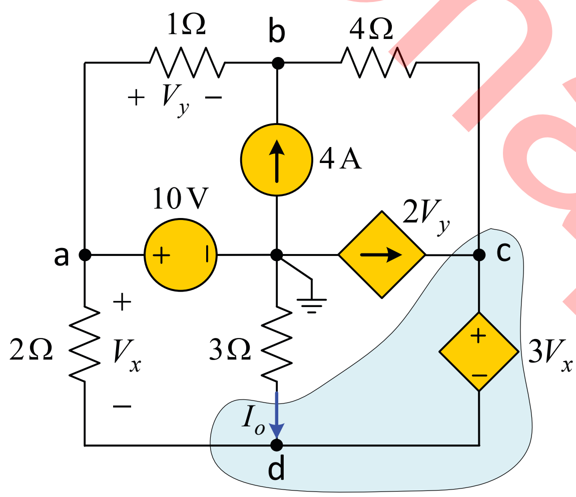

Figure 2

The node equations at node b and at supernode are

$$\text{supernode:}\quad \frac{V_b - V_c}{4} + 2V_y + \frac{-V_d}{3} + \frac{10 - V_d}{2} = 0 \qquad (3)$$

$$\text{node b:}\quad \frac{V_c - V_b}{4} + 4 + \frac{10 - V_b}{1} = 0 \qquad (4)$$

We also have

$$V_x = 10 - V_d \qquad (5)$$

$$V_y = 10 - V_b \qquad (6)$$

Substituting V_x, V_y into equations (1), (3) and (4) we lead to the following system

$$\begin{bmatrix} 0 & 1 & 2 \\ -\frac{7}{4} & -\frac{1}{4} & -\frac{5}{6} \\ -\frac{5}{4} & \frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 30 \\ -25 \\ -14 \end{bmatrix} \qquad (7)$$

From the above system we get

$$V_b = \frac{38}{11}\text{V}, \; V_c = -\frac{426}{11}\text{V} \;\; \text{and} \;\; V_d = \frac{378}{11}\text{V} \qquad (8)$$

Thus,

$$I_o = -\frac{V_d}{3} = -\frac{126}{11}\text{A} = -11.455\,\text{A} \qquad (9)$$